Perturbation dynamics in laminar and turbulent flows. Initial value problem analysis

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15th of April 2014, University of Cagliari, Cagliari, Italy

Introduction

- Perturbations: small variations, externally induced, of one or more physical quantities related to a given physical system.
- The evolution of the spatial-temporal perturbations is a topic of interest for most physical systems. From a physical point of view, in fact, disturbances are always present in reality and can not be eliminated or ignored, whether or not they are infinitesimal.

Study of disturbances spatial-temporal evolution in the the main flow regimes: laminar, in transition and turbulent.

Laminar flows

Perturbation dynamics $\longrightarrow$ Hydrodynamic stability

When...an infinitely small variation of the present state will alter only by an infinitely small quantity the state at some future time, the condition of the system, whether at rest or in motion, is said to be stable; but when an infinitely small variation in the present state may bring about a finite difference in the state of the system in a finite time, the condition of the system is said to be unstable.[Clerk Maxwell]

Hydrodynamic stability is an important part of fluid mechanics, because an unstable flow is not observable

Reynolds experiment

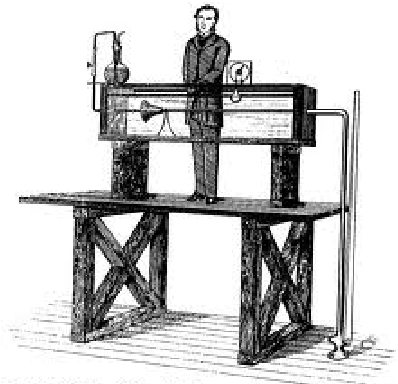


Fig. 9.1. Sketch of Reynolds's dye experiment, taken from his 1883 paper

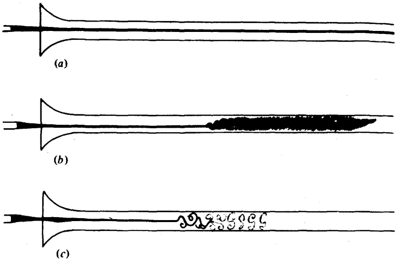
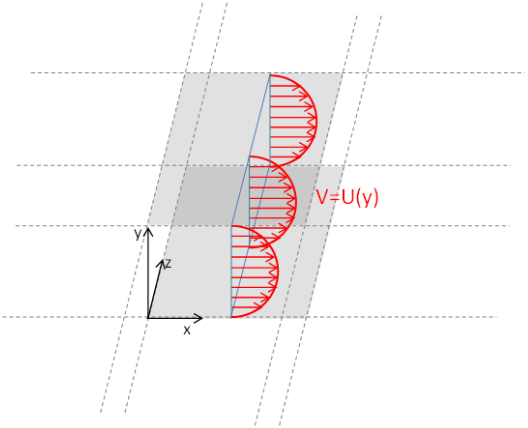


Fig. 9.2. Reynolds's drawings of the flow in his dye experiment.

$$Re = \frac{UL}{\nu} = \frac{\text{inertial}}{\text{visous}}$$

Traditional stability analysis

- Plane Channel flow
- Incompressible
- Constant density
- Two-dimensional
- Parallel



Base flow perturbed by three-dimensional small amplitude disturbances $\longrightarrow$ Linearised NS equations

Linearised Equations

$$\begin{cases} (\partial_t + i\alpha U)(\partial_{yy}\hat{v} - k^2\hat{v}) - i\alpha U''\hat{v} = \dfrac{1}{Re}(\partial_{yyyy}\hat{v} - 2k^2\partial_{yy}\hat{v} + k^4\hat{v}) \\ (\partial_t + i\alpha U)\hat{\omega}_y + i\beta U'\hat{v} = \dfrac{1}{Re}(\partial_{yy}\hat{\omega}_y - k^2\hat{\omega}_y) \\ \hat{v}(t=0) = v_0, \quad \hat{\omega}_y(t=0) = \omega_{y0} \\ \hat{v}(\pm y_b) = \hat{v}'(\pm y_b) = \hat{\omega}_y(\pm y_b) = 0 \end{cases}$$

$\hat{v}, \hat{\omega}_y$: perturbation transversal velocity and vorticity
U, U', U'' : base flow velocity and vorticity derivatives

Linearised Equations

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$$

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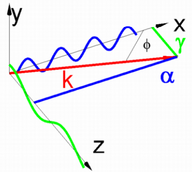
Fourier transform



Modal analysis

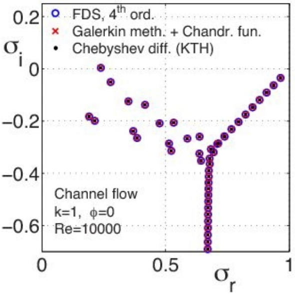
$$-i\sigma \underbrace{\begin{pmatrix} k^2 - \partial_y y & 0 \\ 0 & 1 \end{pmatrix}}_{\mathbf{M}} \begin{pmatrix} \hat{v} \\ \hat{\omega}_y \end{pmatrix} + \underbrace{\begin{pmatrix} \mathcal{L}_{OS} & 0 \\ i\gamma U' & \mathcal{L}_{sq} \end{pmatrix}}_{\mathbf{L}} \begin{pmatrix} \hat{v} \\ \hat{\omega}_y \end{pmatrix} = 0$$

$$\mathcal{L}_{OS} = i\alpha U(k^2 - \partial_y y) + i\alpha U'' + \frac{1}{Re}(k^2 - \partial_{yy})^2$$

$$\mathcal{L}_{SQ} = i\alpha U + \frac{1}{Re}(k^2 - \partial_{yy})$$

$$\mathbf{L}\hat{\mathbf{q}} = i\sigma \mathbf{M}\hat{\mathbf{q}}$$

$$\hat{\mathbf{q}} = \begin{pmatrix} \hat{v} \\ \hat{\omega}_y \end{pmatrix}$$



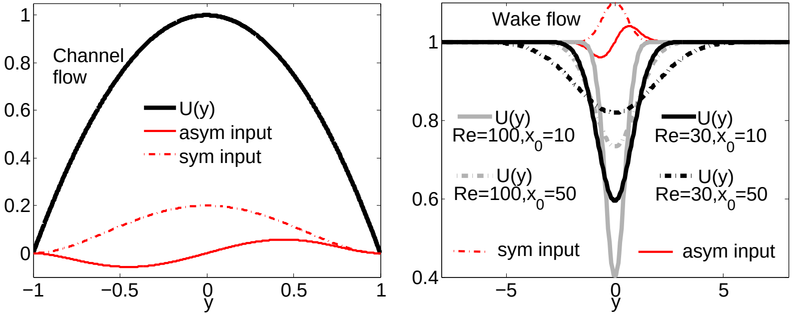
Short term influence

- $\sigma_{i_{max}} < 0$ is not a sufficient condition to guarantees stability $\longrightarrow$ discrepancy between the critical Reynolds number predicted by the modal analysis and the observed transitional Reynolds number.

- For most wall-bounded shear flows the spectrum is a poor proxy for the disturbance behaviour as it only describes the asymptotic fate of the perturbation and fails to capture short-term characteristics$\longrightarrow$Initial Value Problem

- Optimal growth analysis *(Schmid, Annu. Rev. Fluid Mech., 2007)*

Arbitrary Initial Conditions



- Initial conditions: symmetric and asymmetric inputs;
- Boundary conditions: $(\hat{u}, \hat{v}, \hat{w}) \rightarrow 0$ as $y \rightarrow \pm\infty$ and at walls.

Measure of the transient

- Kinetic energy density e:

$$e(t;\alpha,\gamma) \;=\; \frac{1}{2}\int_{-y_d}^{+y_d}(|\hat{u}|^2+|\hat{v}|^2+|\hat{w}|^2)dy$$

- Amplification factor G:

$$G(t;\alpha,\gamma)=\frac{e(t;\alpha,\gamma)}{e(t=0;\alpha,\gamma)}$$

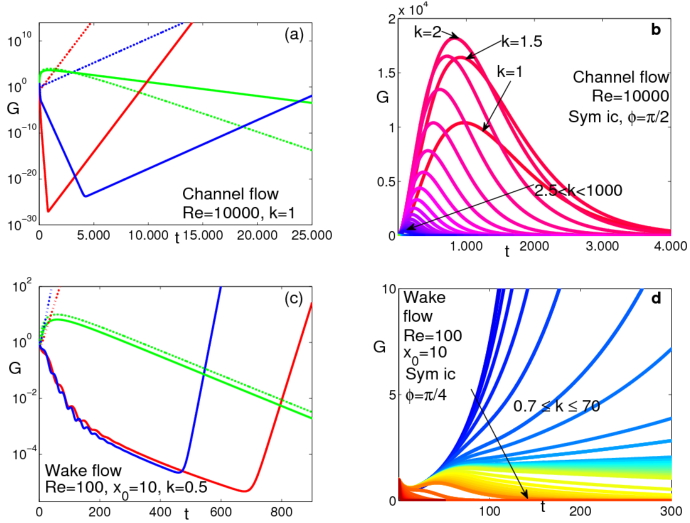
- Temporal growth rate r:

$$r(t;\alpha,\gamma)=\frac{\log(e)}{2t}$$

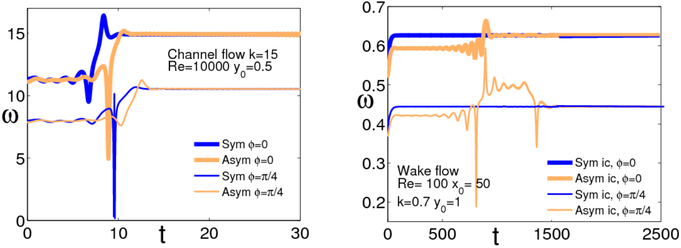
- Angular frequency (pulsation) ω (Whitham, 1974):

$$\omega(t;\alpha,\gamma)=\frac{d\varphi(t)}{dt}, \qquad \varphi \;\; \text{time phase}$$

Amplification factor transient



Frequency Jumps

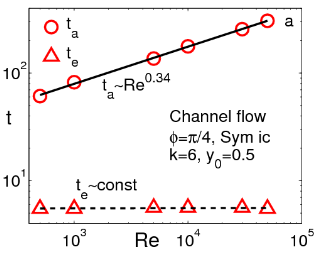


$$T_f = \frac{2\pi}{\sigma_{r_{max}} - \sigma_{r_{min}}}$$

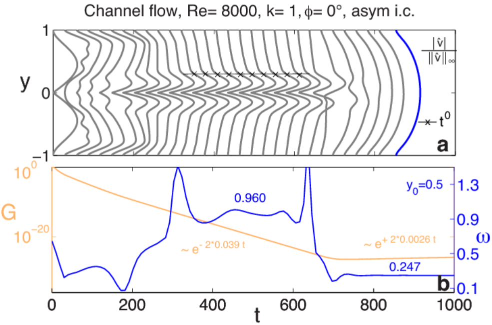
The frequency jump occurs quite far along within the transient and can be considered as the transition from an early transient towards an intermediate transient.

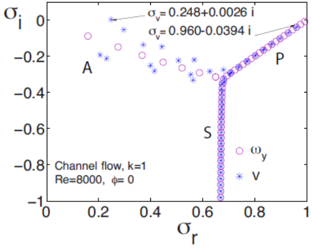
Intermediate Transient definition

- The intermediate term has a duration one order of magnitude longer than the early term $\longrightarrow$ is the most probable state during the life of a perturbation.

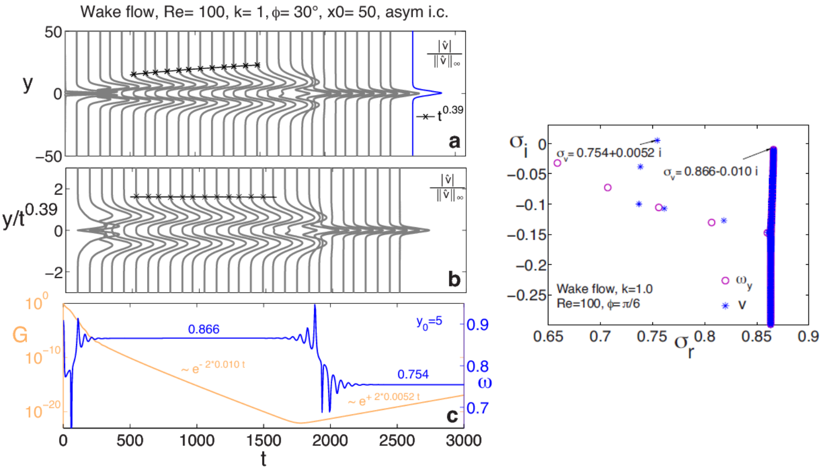


Velocity profile self similarity in channel flow





Velocity profile self similarity in wake flow



Scarsoglio, De Santi, Fraternale, Tordella under review to JFM

Energy Spectra Power-law

- How this velocity scaling property affects the kinetic energy?
- We can consider a system where the base flow is perturbed by many different disturbances $\longrightarrow$ consistence of several different temporal/length scales.
- Leaving aside the non-linear interaction the resulting system has many similarities with turbulent flows

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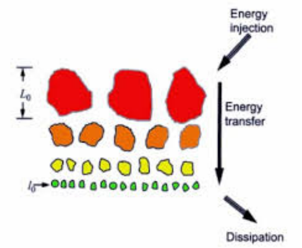
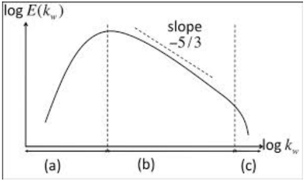




Whether, and to what extent, spectral representation can effectively highlight the non-linear interaction that occurs among different scales?

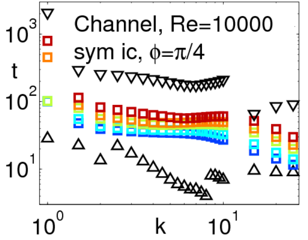
Energy Spectra Power-law within the intermediate transient

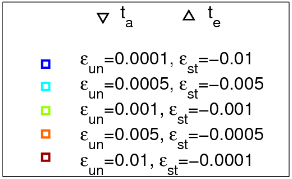
We compare the amplification factor of the waves at a constant value of the energy temporal rate

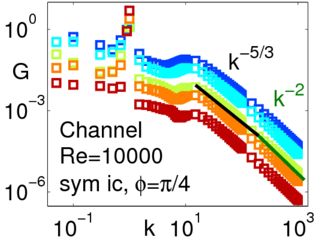
$$|dG/dt| = \varepsilon$$

T_e first frequency jump

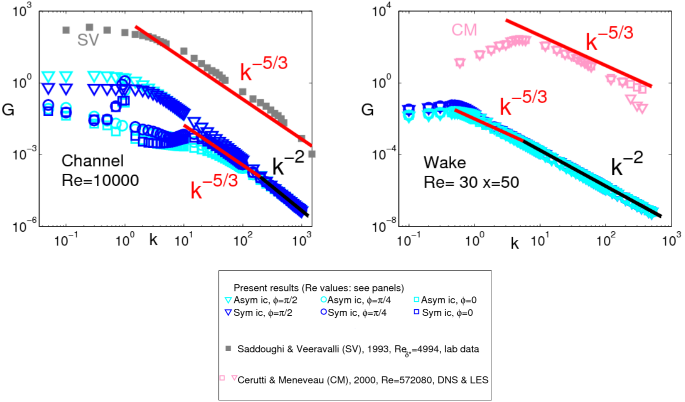
$T_a \rightarrow r = const, \ \omega = const$



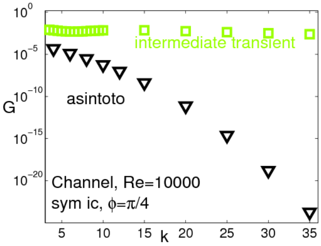


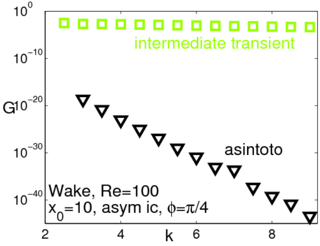


Energy Spectra Power-law within the intermediate transient



Energy Spectra Power-law in the asymptotic regime

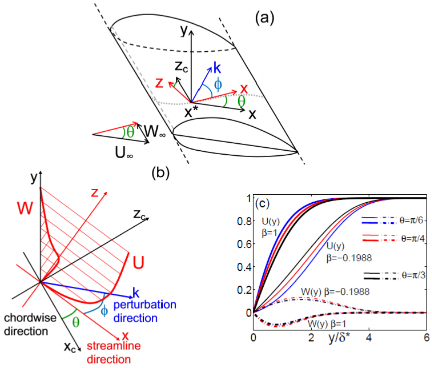




Scarsoglio, De Santi, Tordella Journal of Physics Conference Series 2011

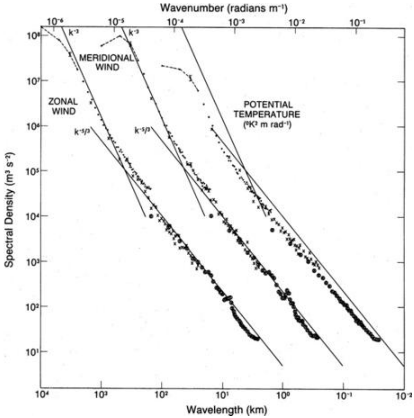
Extension of the results

It can be shown that the results discussed so far are also valid for more complex flows such as the boundary layer in cross flow



De Santi, Scarsoglio, Criminale & Tordella, submitted to Int. J. Heat Fl. Flow

Atmospheric Energy Spectra



Nastrom-Gage (1983) aircraft wind spectra in the upper troposphere

Atmospheric flows properties

Atmospheric flows are strongly influenced by:

- Rotation $\longrightarrow$ Coriolis forces
- Stratification (vertical variation of the fluid density/temperature) $\longrightarrow$ Archimede's force

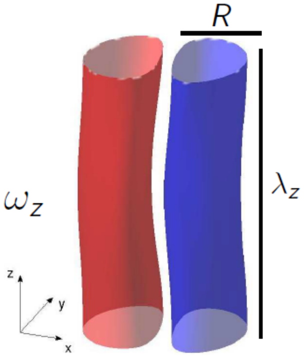
At large scales in the atmosphere rotation becomes of secondary importance and the stratification effects stratification dominate.

Stratified turbulence is highly characterized by anisotropy $\longrightarrow$ Kolmogorov hypothesis are not satisfied

It is possible a better understanding of the stratified turbulence dynamics by considering of a simple flow?

Columnar counter rotating vortices

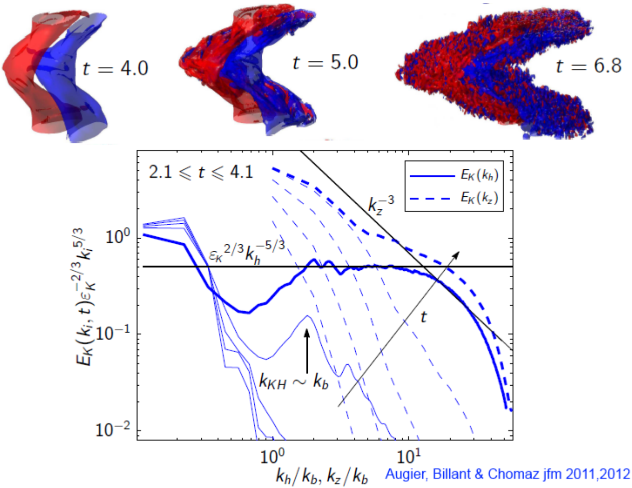
Initial condition:



2D lamb-Chaplygin dipole perturbed by an unstable mode $\lambda_z << R$

- Boussinesq approximation
- Direct Numerical Simulations

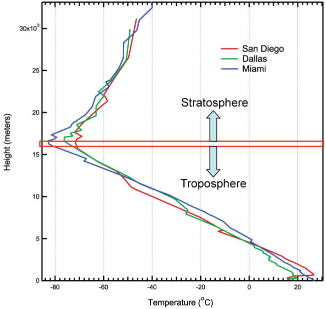
Transition to turbulence

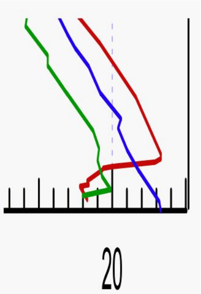


Augier, Billant & Chomaz jfm 2011,2012

Temperature Inversion

- Most of the results concerning the stratified turbulence are obtained a linear stratification profile.

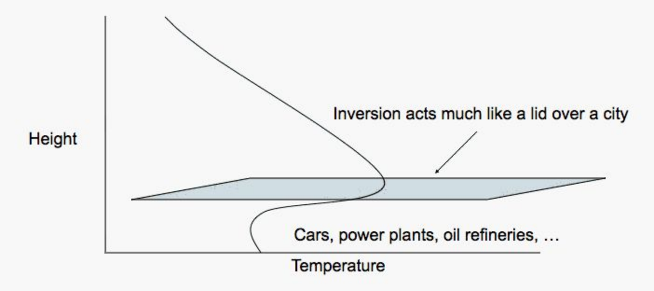




- Atmospheric measurement shows that there are temperature inversion layer

Temperature Inversion

The presence of these inversion layers plays an important role in atmospheric dynamics



Aim: investigate how this kind of temperature profile affects the entrainment at the cloud - quiescent air interface.

Cloud - Clear air interface

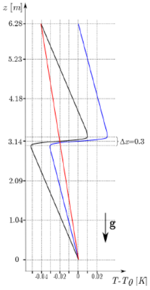


Figure 9: Example of initial conditions for temperature for the case with local temperature gradient equal to $30G$. In blue the profile for θ'

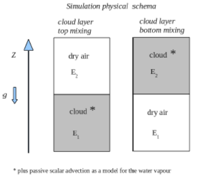
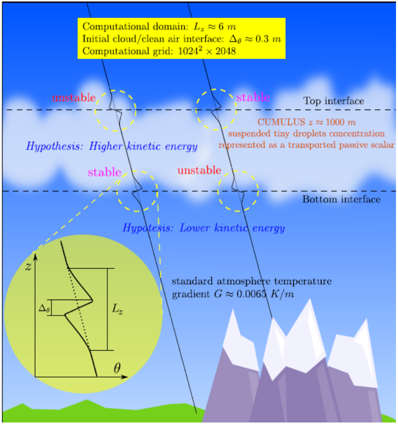
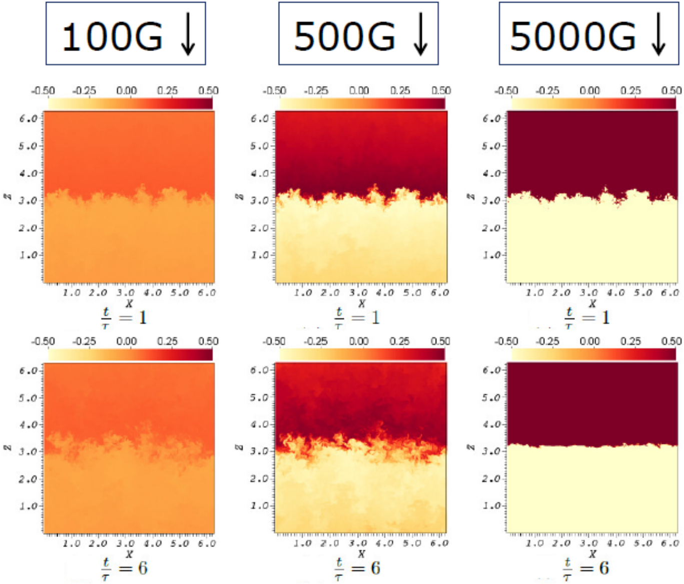


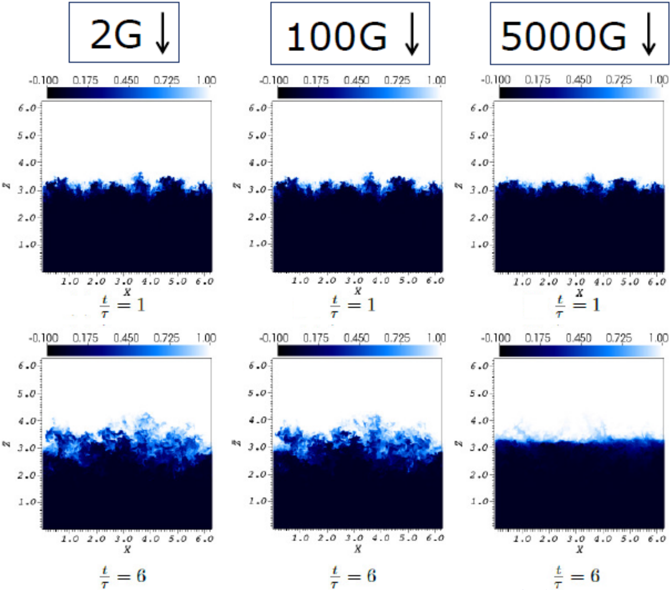
Figure 10: Configuration for the cases of top (left) and bottom (right) mixing. Colored fields indicate the presence of steam in initial conditions.



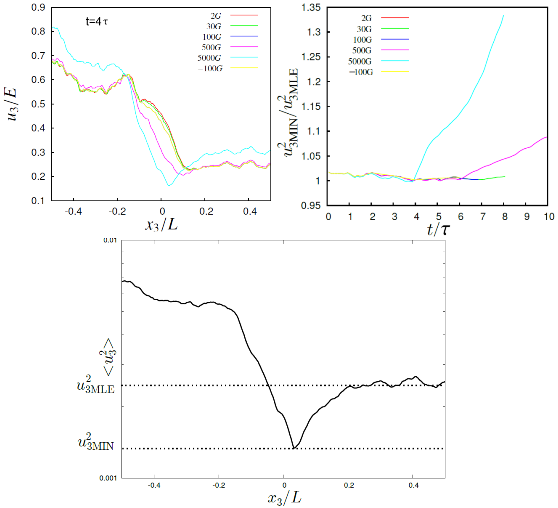
Temperature



Water vapour



Kinetic energy lack at the interface



Concluding Remark

Linear analysis

- Occurrences of frequency jumps $\Rightarrow$ Tripartite structure
- Scaling property inside the intermediate transient
  - self-similarity of the velocity profile
  - power law energy spectrum

Transition to Turbulence

- Stratified turbulence phenomenologies can be well represented by the dynamics of a pair of perturbed columnar counter-rotating vortices

Turbulent mixing

- A temperature inversion layer inhibits the rates of vertical transport of scalars, such as water vapor contained inside the clouds and the turbulent diffusion
- Energy lack at the interface