Two-dimensional shearless turbulent mixing: kinetic energy self diffusion, also in the presence of a stable stratification

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European Postgraduate Fluid Dynamics Conference
9th-12th of August 2011, Politecnico di Torino, Italy

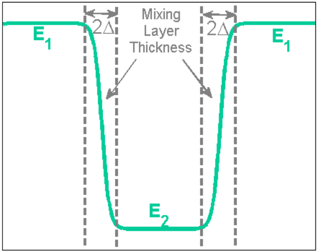
General aspect

- We have performed a numerical experiments concerning the turbulent energy transport
- We have considered the simplest kind of two dimensional turbulent shear-less mixing process
  $\Rightarrow$ the interaction of two isotropic turbulent field with different kinetic energy but the same spectrum shape
- This turbulent transport is observed firstly in a pure shear-less mixing process and in a second time adding the effect of a stable density stratification
  $\Rightarrow$ Conceptual experiment

Numerical Method

- Domain:
  - The computational domain is a $2\pi \times 2\pi$ rectangle with 1024×1024 points
  - Periodic boundary condition in both directions
- Method:
  - Two-dimensional DNS
  - Vorticity stream function formulation
  - Use hyper-viscosity, $\nu = 2.4410^{-9} m^4/s$
  - Solves the Navier Stokes equation by a pseudo-spectral Fourier-Galerkin method, with the $2/3$ de-aliasing technique
  - The time integration is done by a third-step third-order Adams Bashforth method

Flow description



Two decaying turbulent field with Kinetic Energy E_1 and E_2 are matched by means of a hyperbolic tangent function:

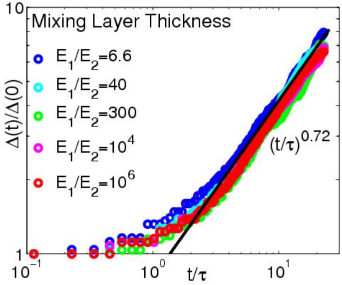
$$u(x) = u_1(x)p(x) + u_2(x)(1 - p(x))$$

$$p(x) = \frac{1}{2}[1 + tanh(a\frac{x}{L})tanh(a\frac{x - L/2}{L})tanh(a\frac{x - L}{L})]$$

(Here $L = 2\pi$ $a = 28\pi$, $\Delta = L/40$)
The ratio of the turbulent kinetic energy has been chosen as the sole control parameter. In particular, the following values of energy ratio were chosen,
$E_1/E_2 = 6.6 \quad 40 \quad 40 \quad 300 \quad 10^4 \quad 10^6$

Time evolution of vorticity contours example

2D - 3D Comparison



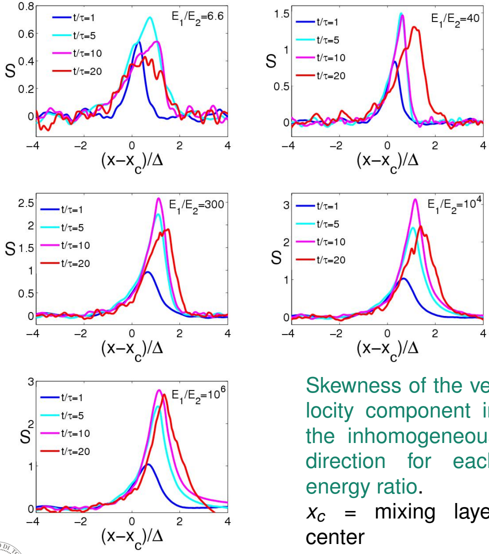
⇒ We define a penetration as the position of the maximum of the skewness normalized over the mixing layer thickness $\eta = \frac{x_s}{\Delta t/\tau}$ and the diffusion velocity $v_{\mathcal{D}} = \frac{dx_s}{dt} = \eta \frac{d\Delta}{dt}$

$$2D : \frac{\Delta(t)}{\Delta(0)} \propto \frac{t^{0.72}}{\tau} \qquad 3D : \frac{\Delta(t)}{\Delta(0)} \propto \frac{t^{0.43}}{\tau}$$

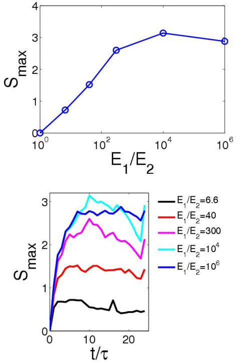
$$v_{\mathcal{D}} = \frac{\eta}{\tau}\frac{d(t^{0.72}/\tau)t}{dt} \propto t^{-0.28} \qquad v_{\mathcal{D}} = \frac{\eta}{\tau}\frac{d(t^{0.43}/\tau)t}{dt} \propto t^{-0.57}$$

In 2D the turbulent diffusion is infinitely grater than the one measured in 3D⇒*movie*

Skewness

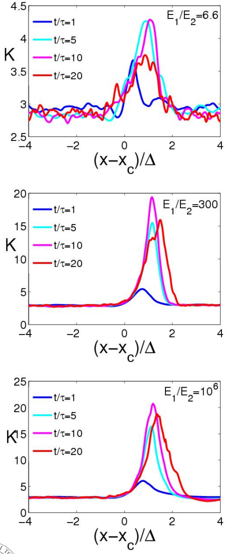


Skewness of the velocity component in the inhomogeneous direction for each energy ratio.
x_c = mixing layer center



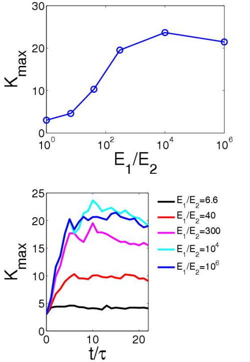
Maximum of the Skewness as a function of the energy ratio and of the time

Kurtosis



Kurtosis of the velocity component in the inhomogeneous direction for each energy ratio.

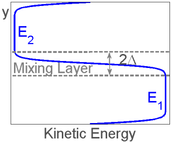
x_c = mixing layer center

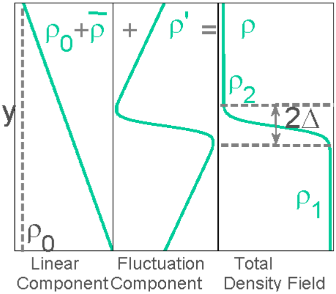


Maximum of the kurtosis as a function of the energy ratio and of the time

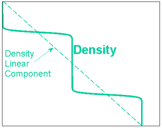
Flow Description

- We change the experiment by adding the effect of a stable stratification
- We rotate the vorticity field
- We create an initial density field by combining two constant density fields with the same hyperbolic tangent used for the vorticity field





- The fluctuation component has periodic boundary condition $\Rightarrow$ The stability of the stratification is guaranteed



- The results obtained in this way can be considered as the vertical section af a three-dimensional stratified flow

Formulation

Using the Boussunesq approximation the equations that describe the problem are:

$$\nabla \cdot \mathbf{u} = \mathbf{0}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\mathbf{1}}{\rho_{\mathbf{0}}}\nabla \mathbf{p} - \frac{\rho'}{\rho_{\mathbf{0}}}\mathbf{g} + \nu\,\acute{}\,\mathbf{u}$$

$$\frac{\partial \rho'}{\partial t} + (\mathbf{u} \cdot \nabla)\rho' + \mathbf{v}\frac{\mathbf{d}\rho_{\mathbf{m}}}{\mathbf{dy}} = \mathbf{k}\,\acute{}\,\mathbf{u}$$

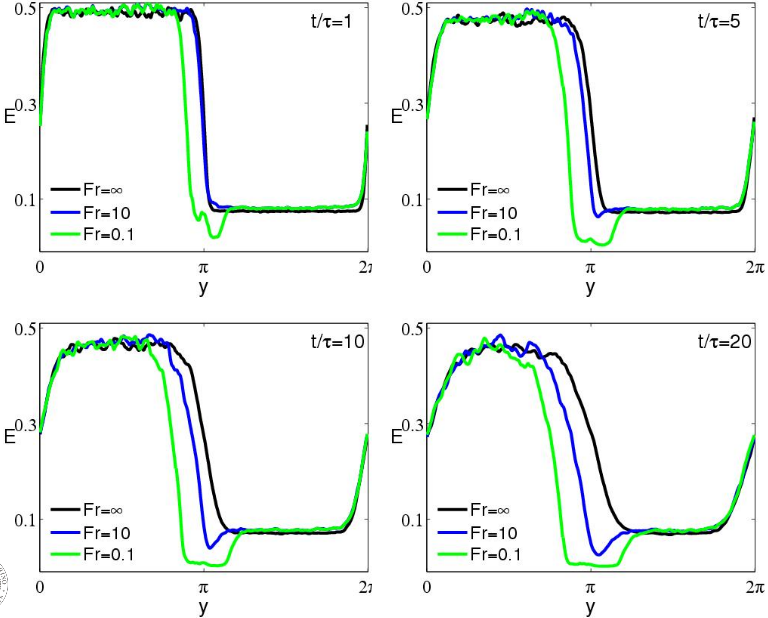
$$\nu = 2.4\ 10^{-10} m^4/s,\ k = 0.3\ 10^{-2},\ Sc* = (\nu/(k * l^2)) = 1.32\ 10^{-4}$$

- The energy ratio is constant, $E_1/E_2 = 6.6$
- The parameter of the experiment is the Froude number

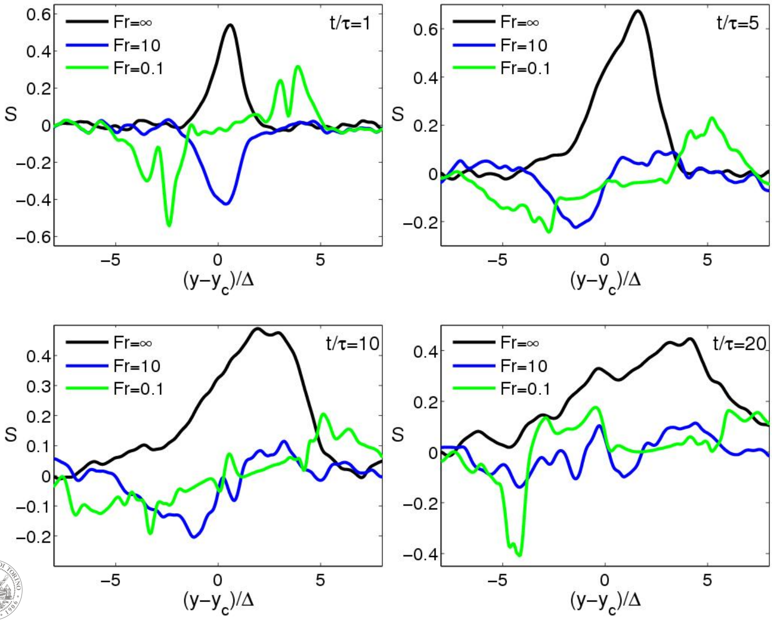
$$Fr = \frac{U}{\sqrt{-\frac{g}{rho_0}\frac{\partial \rho_m}{\partial y}}L}$$

$Fr = \infty$, $Fr = 10$, $Fr = 0.1$ *movie*

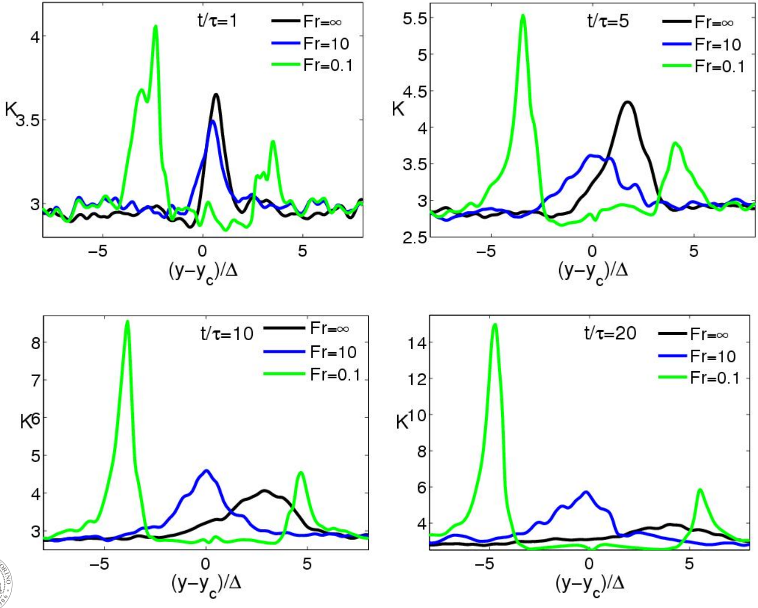
Kinetic Energy Profile



Skewness



Kurtosis


t/τ=1
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=5
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=10
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=20
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ

Conclusion 1

Experiment 1: Interaction between two isotropic turbulent filed with different kinetic energy but the same spectrum shape

- The turbulent diffusion is infinitely greater than the one measured in 3D
- The analysis of the velocity in the inhomogeneous direction indicates that the flow is highly intermittent ⇒ Intermittency front
- The flow presents a long-range interaction

Experiment 2: Interaction between two isotropic turbulent filed with different kinetic energy and density but the same spectrum shape

- For small Froude numbers it is formed a separation layer of zero vorticity
- The energy profile in the mixing region is lower than the minimum value imposed by the initial condition, which shows the effect of the buoyancy force work ⇒ Energy hole
- The velocity skewness enlightens the generation of an inverse energy flow and intermittent penetration from the low to the high energy field even in the case of mild stratification

References

[1] C. Canuto, M.Y. Hussaini, A. Quarteroni, and T.A. Zang. *Spectral method. Fundamentals in single domains.* Springerl, 2006.

[2] P.A. Davidson. *An introduction to magnetohydrodynamics.* Cambridge university press, 1988.

[3] U. Frish. *Turbulence: The legacy of A.N.Kolmogorov.* Cambridge University Press, 1995.

[4] U. Frish and G. Parisi. A multifractal model of intermittency. in turbulence and predictability in geophisical fluid dynamics and climate dynamics. 1985.

[5] J. Hinze. *Turbulence.* McGraw-Hill, 1959.

[6] H. Kellay and W. Goldburg. Two-dimensional turbulence: a review of some recent experiment. *Rep. Prog. Phys.*, 65, 2002.

[7] A. Kolmogorov. The local structure of turbulence in incompressible viscous fluid flow for very large reynolds number. *Dokl. Akad. Nauk*, 26, 1941.

[8] A. Kolmogorov. A refinement of previous hypotesis concerning the local structure of turbulence in incompressible viscous fluid at high reynolds number. *J. Fluid Mech*, 13, 1962.

[9] P. Kundu and I. Cohen. *Fluid Mechanics.* Academic press, 2002.

[10] L. Landau and E. Lifshitz. *Fluid mechanics 2nd ed.* 1987.

[11] L. Richardson. *Weather prediction by numerical process.* Cambidge University Press, 1992.

[12] J. Riley and S.M. deBruynKops. Dynamics of turbulence strongly influenced by buoyancy. *Phys. Fluids*, 15, 2003.

[13] J. Riley and M.P. Lelong. Fluid motions in the presence of strong stable stratification. *Ann. Rev. Fluid Mech.*, 32, 2000.

[14] D. Tordella and M. Iovieno. Numerical experiments on the intermediate asymptotics of shear-free turbulent transport and diffusion. *J.Fluid Mech.*, 549, 2006.

[15] D. Tordella, M. Iovieno, and P.R. Bailey. Sufficient condition for gaussian departure in turbulence. *Phys.Rev.*, 77, 2008.

[16] D.J. Tritton. *Physical fluid dynamics.* Oxford science publications, 2006.

[17] S. Veeravalli and Z. Warhaft. The shearless turbulence mixing layer. *J. Fluid. Mech.*, 207, 1989.