Cumulative distribution of the stretching of vortical structures in isotropic turbulence

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By using a Navier-Stokes isotropic turbulent field numerically simulated in the box with a discretization of 1024^3 [Biferale et al. (2005)], we show that the probability of having a stretching-tilting larger than twice the local enstrophy is very small. This probability decreases if we try to filter out the large scales, while it increases filtering out the small scales. This is basically due to the suppression of the compact structures (blobs).

The normalized stretching-tilting function

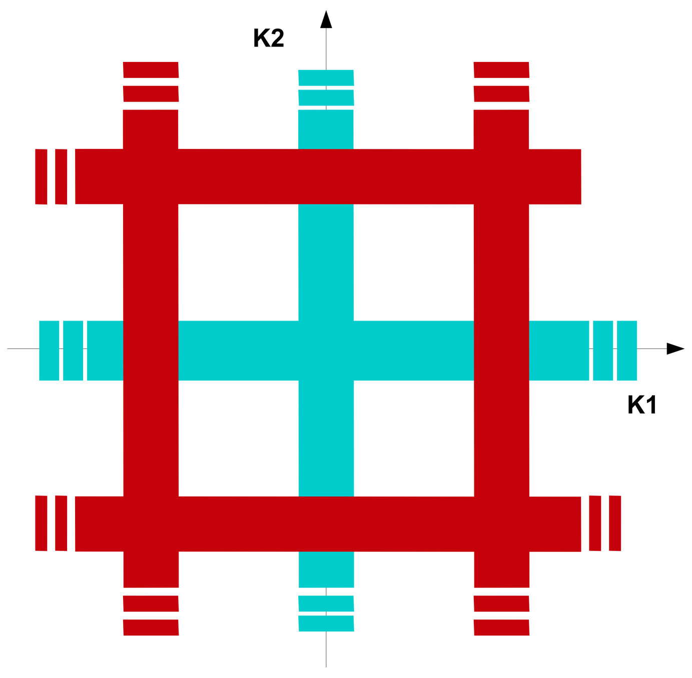
With reference to the phenomena described by the inertial nonlinear nonconvective part of the vorticity transport equation, let us introduce a local measure of the process of three-dimensional inner scales formation [***D. Tordella, M. Iovieno, M. Massaglia (2007-2008)***]:

$$f(\mathbf{x}) = \frac{|\boldsymbol{\omega} \cdot \nabla \mathbf{u}|}{|\boldsymbol{\omega}|^2}(\mathbf{x})$$

The numerator is the so called stretching-tilting term of the vorticity equation. In three-dimensional fields, it is responsible for the transfer of the kinetic energy from larger to smaller scales (positive or extensional stretching) and vice-versa (negative or compressional stretching) [***A. Tsinober, E. Kit, T. Dracos (1992), Andreapoulos, Y. & Honkan, A. (2001)***].

Properties of the survival function of the normalized stretching-tilting: analysis on filtered fields

The application of filters to the velocity field, carried out in the wavenumber space by means of suitable convolutions, allows to analyse the behaviour of the function f(x) in the different scale ranges of the turbulence. The analysis of the behaviour of the function f(x), in different scale ranges of the turbulence is mainly performed by using two kinds of filters, a high pass filter and a band-cut filter.



Scheme of the cross filter in the k_1 k_2 wavenumber plane that can be used as i) a high-pass filter: the wave-numbers under a certain threshold are partially removed (region in blue), ii) band-cut filter: the wave-numbers inside a range, or above a certain threshold, are cut (region in red).
In the first one is essentially a cut-off filter, which we refer to as cross filter and which allows the contribution of the structures that are characterized by at least one large dimension to be removed. In the Fourier space, this means:

$$k_1 < k_{MAX} \;\text{ or }\; k_2 < k_{MAX} \;\text{ or }\; k_3 < k_{MAX}.$$

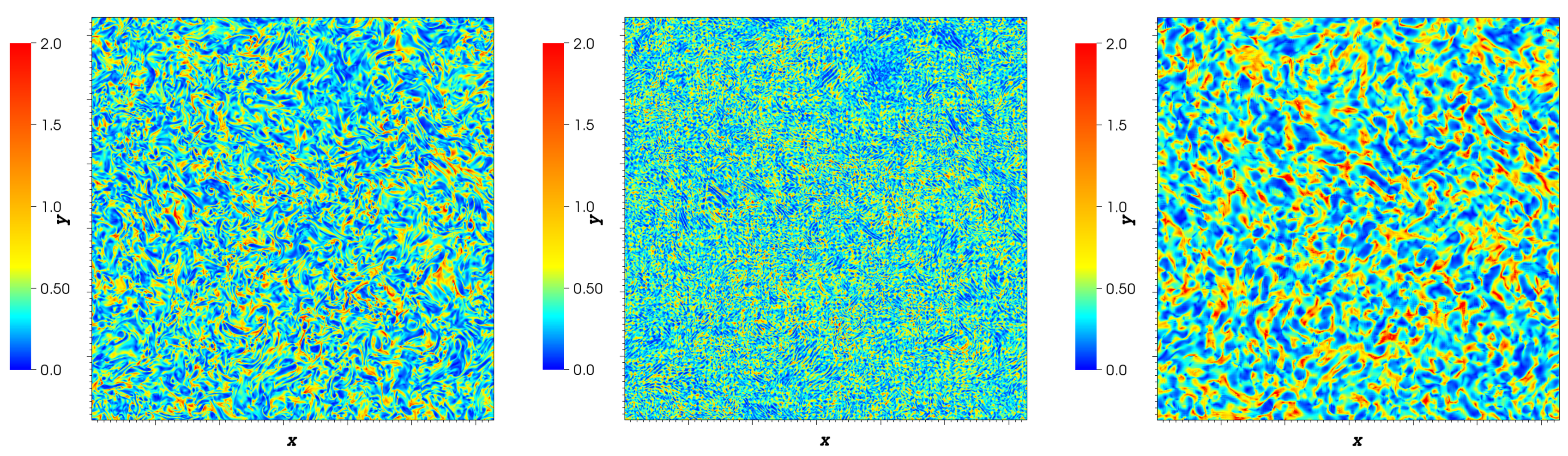
The representation of this high-pass filter in Fourier space can be given by the following function [***Tordella & Iovieno (2006)***]:

$$g_{hp}(\underline{k}) = \prod_i \phi(k_i),\; \phi(k_i) = \frac{1}{1+e^{-(k_i - k_{MAX})}}.$$

The second filter can be obtained by reducing the contribution of a variable band:

$$k_{MIN} < k_1 < k_{MAX} \;\text{ or }\; k_{MIN} < k_2 < k_{MAX} \;\text{ or }\; k_{MIN} < k_3 < k_{MAX}.$$

If the high pass filter is used, the value of the function f is reduced (bottom left panel), while, if the band-cut filter is used, the function grows up (bottom right panel).
The cross filter here used does not spoil the self-similarity of the field.



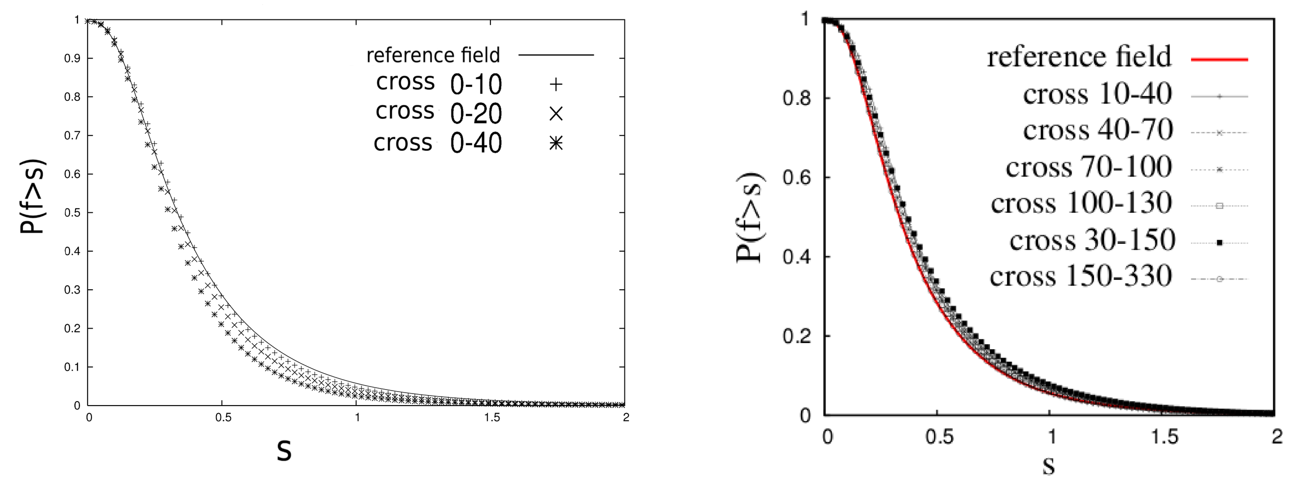
Visualization of the values of function f(x) in a plane (a two-dimensional section of the cubic domain, 1024^3 grid points). Left: reference field. Center: the wave number range 0-20 is filtered out. Right: the wave number range 30-150 is filtered out.

The reduction of the survival function:

$$S(s) = P(f(x) > s) = 1 - F(s)$$

grows up as the threshold K_{MAX} of the high pass filter increases.
A different behaviour can be expected when we try to filter smaller scales, by applying the band-cut filter to the inertial range of scales. ***(P.E. Hamlington, J. Schumacher and Dahm (2008)).***

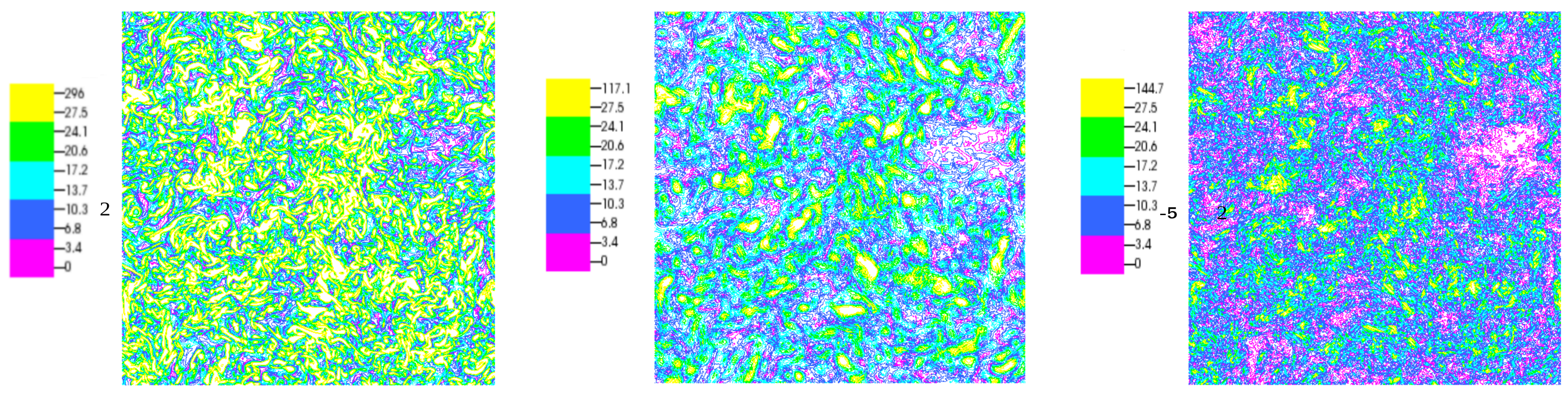
The probability that f(x) > 2 is almost zero. Thus, f (x) = 2 can be considered the maximum statistical value that f(x) can reach when the isotropic turbulence is simulated.



Probability of the normalized stretching-tilting function in a band pass filter and a high pass filter (in the inertial range) isotropic turbulent field of being higher than a threshold s [***D. Tordella, M. Iovieno, M. Massaglia (2007-2008)***].

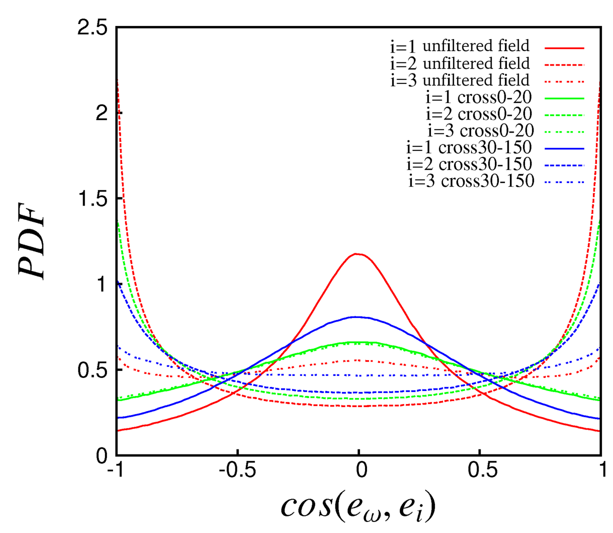
Filter effect on the vorticity field

The filters also modify the structures of vorticity. It is used the 30-150 band cut filter and a 0 − 20 high pass filter. The first one reduces the small structures (blobls, sheets and tubes) of a component of the vorticity. The second one removes large structures (blobls, sheets and tubes) of a component of the vorticity.



Isotropic turbulence. Visualization of the adimensional values of countor plot of vorticity magnitude, in a section of the domain, 256^2 grid point at Re_λ=280, u_{rms} = 0.03684 m/sec (root-mean-square velocity), ν=1.5x10^{-5} m^2/sec (viscosity) and L = 2m (integral scale).
Lines outline the iso-surphace contours. Top: unfiltered field. Middle: the wave number range 0-20 is filtered out. Bottom: the wave number range 30-150 is filtered out.

Isotropic Turbulence. Visualization of a component of vorticity, in a three-dimensional cubic domain, 256^3 grid points at Re_λ= 280, u_{rms} = 0.03684 m/sec (root-mean-square velocity), ν= 1.5x10^{-5} m^2/sec (viscosity) and L = 2m (integral scale). The value of the field is adimensional and is equal to 18. Left: unfiltered field. Center: the wave number range 30-150 is filtered out. Right: the wave number range 30-150 is filtered out.



PDFs of the cosine of the angle between vorticity, ω, and the eigenframe λ_i of the rate strain tensor. In all the three cases there exists a strong alignment between ω and λ_2 [***A. Tsinober, An Informal Introduction to Turbulence, 2001***]. The filter 0-20 does not induces substancial differencies of PDF values for λ_1 and λ_2 . The effect of the filters induces lower PDF values of the alignment for all three eigenvalues (λ_1,λ_2,λ_3).

CONCLUSIONS:

- The probability of having a stretching/tilting of intensity larger than twice the square of the vorticity magnitude is almost zero [*D. Tordella, M. Iovieno, M. Massaglia (2007), see D. Tordella, M. Iovieno, M. Massaglia (2008)*]

- When compact structures in the inertial range are filtered out, the probability of having f(x) higher than a given threshold, s, increases by 20% at s = 0.5, and by 60-70% at s = 1. If larger blobs are instead filtered, an opposite situation occurs. The present observations must be associated to the non discriminating effect of filtering on filaments and sheets, which is due to their specific nature that cannot be reconciled inside either a category of small or large scales.

- The cross filtering used does not spoil the self-similarity of the field f(x). A review version of this work is in preparetion for Physical Review E.