

Vorticity fluctuation in the LES of the channel flow through new wall conditions and the non commutation procedure.

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1 Introduction

A set of LES results concerning the vorticity fluctuation in a turbulent channel flow is here presented. Vorticity fluctuation data are rare in literature. Up to now, streamwise vorticity fluctuation has only been measured in one channel flow experiment [7]. In the field of numerical simulations, the vector of the vorticity fluctuations has only been determined in three direct simulations [1,8,10]. At the state of the art of large eddy simulations, such quantities have never appeared before.

The vorticity fluctuations across a channel were here obtained using a new type of wall treatment for resolved large scale simulations that extend inside the viscous sublayer [4]. There are two points at issue in this treatment: a - the transfer of the no-slip and impermeability/permeability wall condition (which are normally only applicable to unfiltered variables) to filtered variables and b - the explicit treatment of the non commutation property loss between the filter and the differentiation operations, which affects the simulation of inhomogeneous fields where the filter scale varies ($\delta = \delta(x_i)$). In this situation the governing equations change structure, as a noncommutation term must be introduced in correspondance to each spatial differential term. The treatment consists of the use of new wall boundary conditions and of the explicit noncommutation procedure proposed by Iovieno and Tordella(2003)[5]. It improves the use of the large eddy method in relation to aspects that are independent of the modeling of the subgrid scale motion. When applied to the plane periodic channel case intentionally using the most crude subgrid scale model (Smagorinsky, with no dynamic procedure or wall damping function) to prove its efficacy, the proposed near-wall treatment yielded resolved large-eddy simulations which compare well with both direct numerical simulations and with experimental data [4]. The new conditions transfer the physical no-slip and impermeability/permeability information to the filtered variables on an infield boundary that is placed about one viscous length from the wall.

2 Method and discussion on the results

Due to the closeness of the infield condition to the wall, it is possible to transfer the information that is relevant to the physical wall to the shifted condition through local expansions. The transfer is accomplished by considering δ series expansions for the filtered variable at the shifted boundary. If associated to a Taylor expansion of the unfiltered variable at the wall, this yields a first kind of condition that is universal in character (condition **I**). If the δ expansion is instead related to a Mac Laurin expansion of the unfiltered variable at the wall, a second kind of boundary condition is obtained which makes it suitable to impose known distributions of wall stresses, as can happen in inverse mathematical problems (condition **II**). For formulation **I** this yields

$$\langle f \rangle_{sb} = f(0) + \underline{y} \partial_y \langle f \rangle + \frac{1}{2} a \nabla^2 [\langle f \rangle] \delta_{sb}^2 - \frac{y^2}{2} \partial_y^2 \langle f \rangle \quad (1)$$

where $f = u_i$, $a = \int \eta^2 d\eta$, and $sb =$ the shifted boundary, and for formulation **II**

$$\langle f \rangle_{sb} = f(0) + \underline{y} \partial_y f|_{y=0} + \frac{y^2}{2} \partial_y^2 f|_{y=0} + \frac{a}{2} \delta_{min}^2 \nabla^2 [\langle f \rangle]. \quad (2)$$

The other feature that was implemented in the simulation is the non commutation procedure [5]. This is based on an approximation of the different noncommutation terms of the governing equations as functions of the δ gradient and of the δ derivatives of the filtered variables. The anisotropic noncommutation terms, of the fourth order of accuracy in the filter scale, are obtained by using series expansion in the filter width of approximations based on finite differences and by introducing two successive levels of filtering.

A few results concerning the near-wall dynamics of the turbulent channel flow (Re=180 and 590, the boundary condition placed at $y^+ = 2$ and 5) are contrasted in the figures with DNS results [8,9]. Despite the rather unappropriate SGS model that was used and, thus, *a priori* foreseeing a poor agreement in a portion of the viscous sublayer - the systematic error linked to the SGS model, SGSmSE, see figures 1-3 - the agreement is good. This confirms that the present treatment is fruitful. Table 1 gives the L_2 relative error estimates for the turbulence intensities ($5 \leq y^+ \leq 50$, with respect to the filtered DNS data) and the Reynolds stress $\langle uv \rangle$ ($y^+ \geq 20$) for both conditions **I** and **II**. The estimates are comparable with the resolved simulation [3], which takes advantage of the dynamic Smagorinsky model. However, they show a noticeable improvement with respect to the streamwise intensity. Attention should especially be drawn to the good behaviour that is exploited due to the distribution of the vorticity fluctuations described in fig.3. This figure shows, on average, an integral error estimate of the order of 15% with respect to the filtered DNS data, a rather low value for quantities that can be attained with great difficulty both in experiments and in numerical simulations.

In conclusion, this near-wall treatment improves the use of the LES method performance independently of the modelling of the subgrid scale motion. When applied to the plane periodic channel case, intentionally using the most crude subgrid scale model to prove its efficacy, it yields resolved large-eddy simulations which compare well with direct numerical simulations and experiments, even with regards to quantities that are difficult to attain as vorticity fluctuations, which, in fact, are seldom determined in literature. Thus, one can infer that greater improvements can be obtained using more physically adequate models that can partly allow the inverse cascade [2,6] and/or the dynamical procedure to be taken into consideration [3].

—		u'	v'	w'	$\langle uv \rangle$
(I)	180	8.1	1.6	6.1	1.6
(II)	180	16.3	7.8	10.0	8.5
(I)	590	—	—	—	2.5
(II)	590	—	—	—	3.4
Ref. [3]		21.6	1.7	6.6	—

Table 1: L_2 relative error estimates, $y^+ \leq 50$ for the turbulent intensities, $y^+ \geq 20$ for the Reynolds stress.

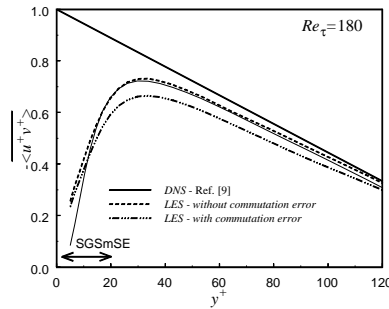


Figure 2: Reynolds stress.

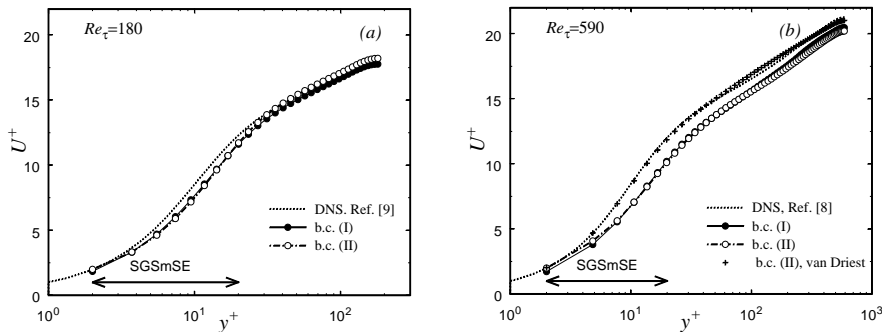


Figure 3: $y^+ < 20$: the slope inaccuracy due to SGSmSE is removable, see text.

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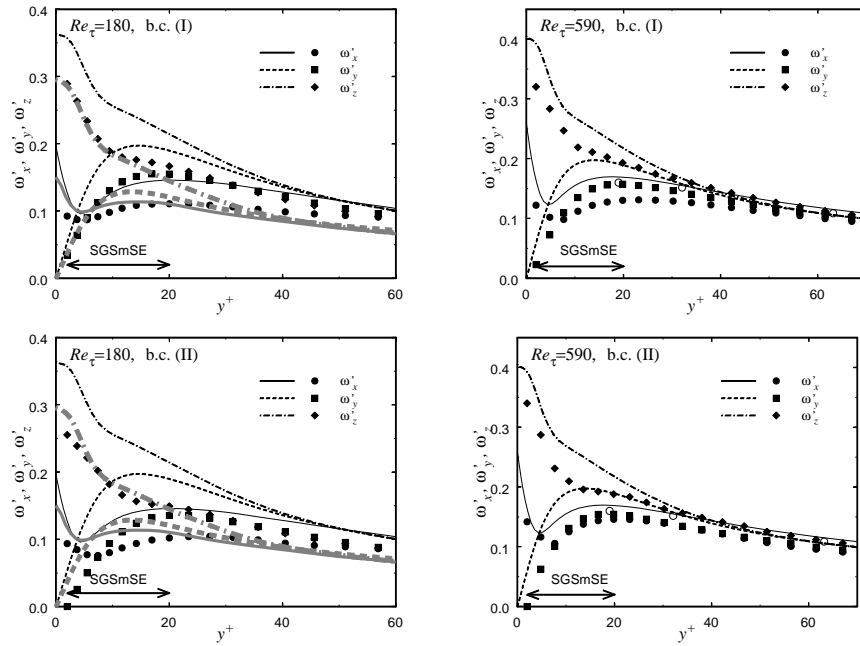


Figure 4: Vorticity rms. Thin lines: unfiltered DNS, grey lines: filtered DNS, filled symbols: present LES, empty symbols: data from Ref. [7].

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