Shearless turbulence mixing: numerical experiments on the intermediate asymptotics

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– D.Tordella, M.Iovieno "Self-similarity of the turbulence mixing with a constant in time macroscale gradient" *22nd IFIP TC 7 Conference on System Modeling and Optimization*, Torino, July 18-22, 2005.

– M.Iovieno, D.Tordella 2002 "The angular momentum for a finite element of a fluid: A new representation and application to turbulent modeling", *Physics of Fluids*, **14**(8), 2673–2682.

– M.Iovieno, C.Cavazzoni, D.Tordella 2001 "A new technique for a parallel dealiased pseudospectral Navier-Stokes code." *Computer Physics Communications*, **141**, 365–374.

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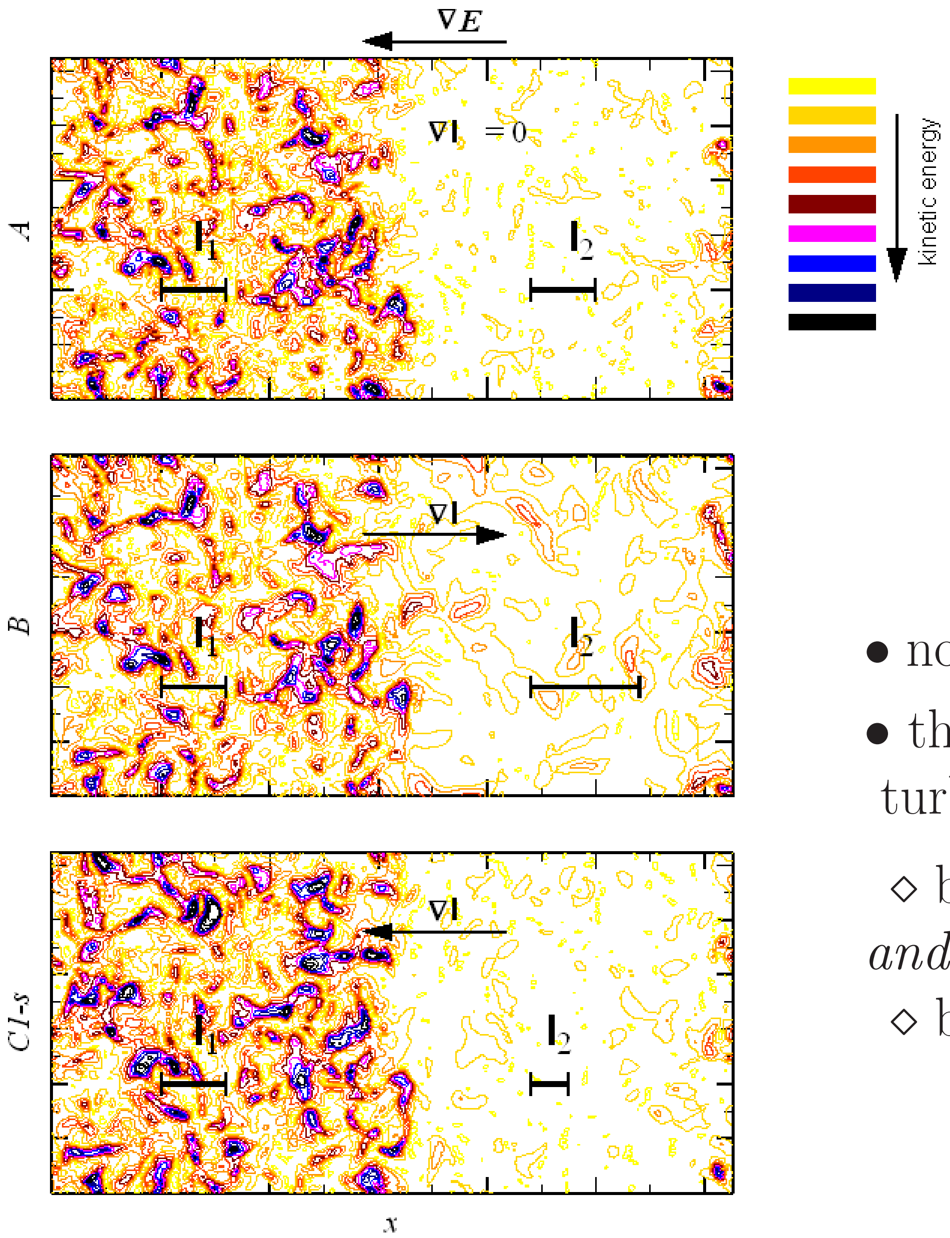
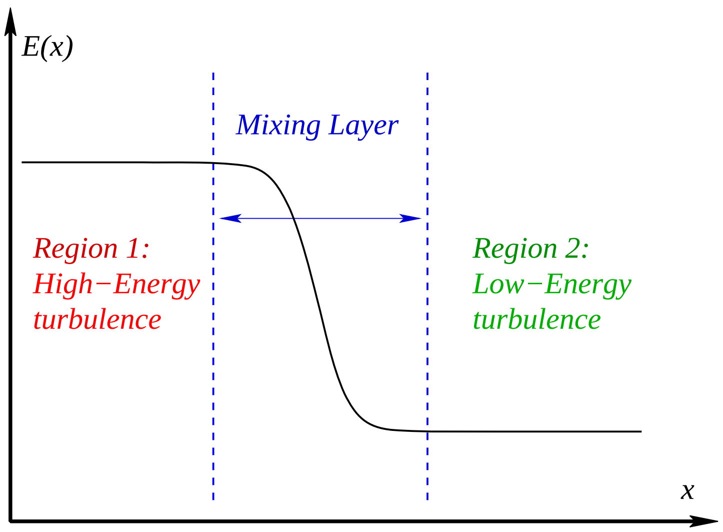
Shearless turbulence mixing.





- no mean shear ⇒ *no turbulence production*
- the mixing layer is generated by the turbulence inhomogeneity, i.e.:

◇ by the gradient of *turbulent energy*

and

◇ by the gradient of *integral scale*

Previous investigations:

Esperiments with grid turbulence:

– Gilbert B. *J. Fluid Mech.* **100**, 349–365 (1980).

– Veeravalli S., Warhaft Z. *J. Fluid Mech.* **207**,191–229 (1989).

Numerical simulations (DNS):

– Briggs D.A., Ferziger J.H., Koseff J.R., Monismith S.G. *J. Fluid Mech.* **310**, 215–241 (1996).

– Knaepen B., Debliquy O., Carati D. *J. Fluid Mech.* **414**, 153–172 (2004).

- in (passive) grid turbulence the higher energy is always associated to larger integral scales, so the two parameters are not independent $\Rightarrow$ *guess about no intermittency in the absence of scale gradient and turbulence production*.
- numerical simulations reproduced the 3,3:1 laboratory experiment by Veeravalli and Warhaft.

New decay properties

- the two parameters, the *turbulent kinetic energy* ratio $\mathcal{E}$ and the *integral scale* ratio $\mathcal{L}$, has been independently varied
- the persistency of intermittency in the limit of no scale gradient ($\mathcal{L} \to 1$) and absence of turbulence production has been investigated.

In particular we present:

- *Part 1:* results from numerical simulations (DNS and LES, 2005 JFM, to appear)
- *Part 2:* intermediate asymptotics analysis ($\mathcal{L} \to 1$, 2005 IFIP TC7 and DLES6; $\mathcal{L} \neq 1$, in preparation)

Part 1: numerical experiments

Numerical simulations (DNS and LES) have been carried out with

- Fixed energy ratio $\mathcal{E} \sim 6.7$ and varying scale ratio $0.38 \leq \mathcal{L} \leq 2.7$
- No scale gradient ($\mathcal{L} = 1$) and variable energy ratio $1 \leq \mathcal{E} \leq 58.3$

- Reynolds number: $Re_\lambda \approx 45$ (DNS, LES) and $Re_\lambda \approx 450$ (LES only, IAM model, Iovieno & Tordella *Phys.Fluids* 2002)

- Numerical method: Fourier-Galerkin pseudospectral on a $2\pi^3$ cube and a $2\pi \times 2\pi \times 4\pi$ parallelepiped (Iovieno-Cavazzoni-Tordella *Comp.Phys.Comm.* 2001)
  Resolution: DNS = $128^2 \times 256$, LES = $32^2 \times 64$
- Initial conditions: two turbulent fields coming from simulations of decaying homogeneous isotropic turbulence.

Decay exponents

- The two homogeneous fields decay algebrically in time, according to theoretical (and experimental) results (see Karman and Howarth 1938, Sedov 1944, Batchelor 1953, Speziale 1995)

$$E = A(t + t_0)^{-n}$$

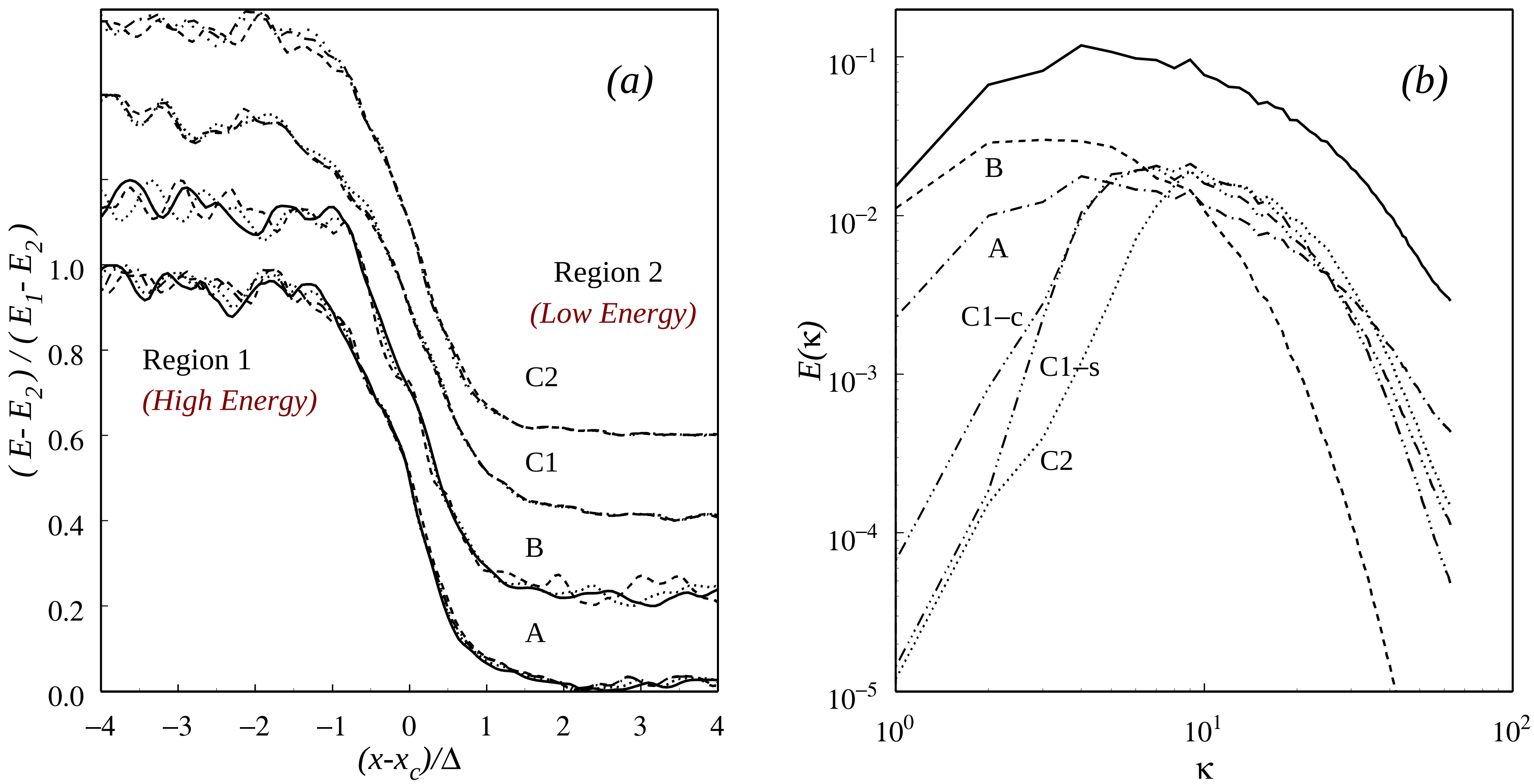
- Decay rates n_1, n_2 are higher than the limit, $n = 1$, for high Reynolds number, but still close to this value ($n_1 \approx n_2 \approx 1.2 - 1.4$), so that the energy and scale ratios remain nearly constant (up to 10%) during the decay

$$\frac{\mathcal{L}(t)}{\mathcal{L}(0)} = \left(1 + \frac{t}{t_{01}}\right)^{1 - \frac{n_1}{2}} \left(1 + \frac{t}{t_{02}}\right)^{-1 + \frac{n_2}{2}}$$

$$\frac{\mathcal{E}(t)}{\mathcal{E}(0)} = \left(1 + \frac{t}{t_{02}}\right)^{n_2} \left(1 + \frac{t}{t_{01}}\right)^{-n_1}$$

- All mixings have an intermediate self-similar stage of decay

Energy similarity profiles

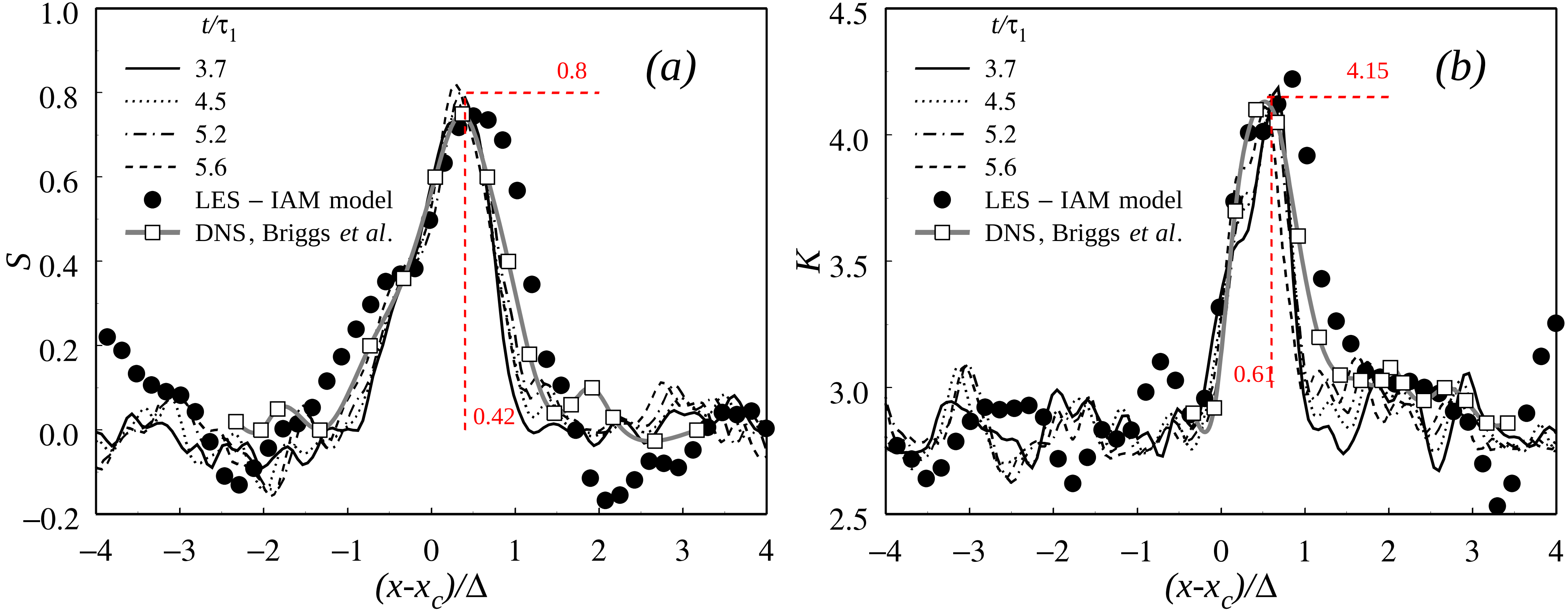


$\Delta(t)$ = mixing layer thickness, $\ell(t) = \frac{1}{3}\Sigma_i \frac{\int_0^\infty R_{ii}(r,t)dr}{R_{ii}(0,t)}$, where R_{ii} is the longitudinal velocity correlation (see e.g. Batchelor, 1953).

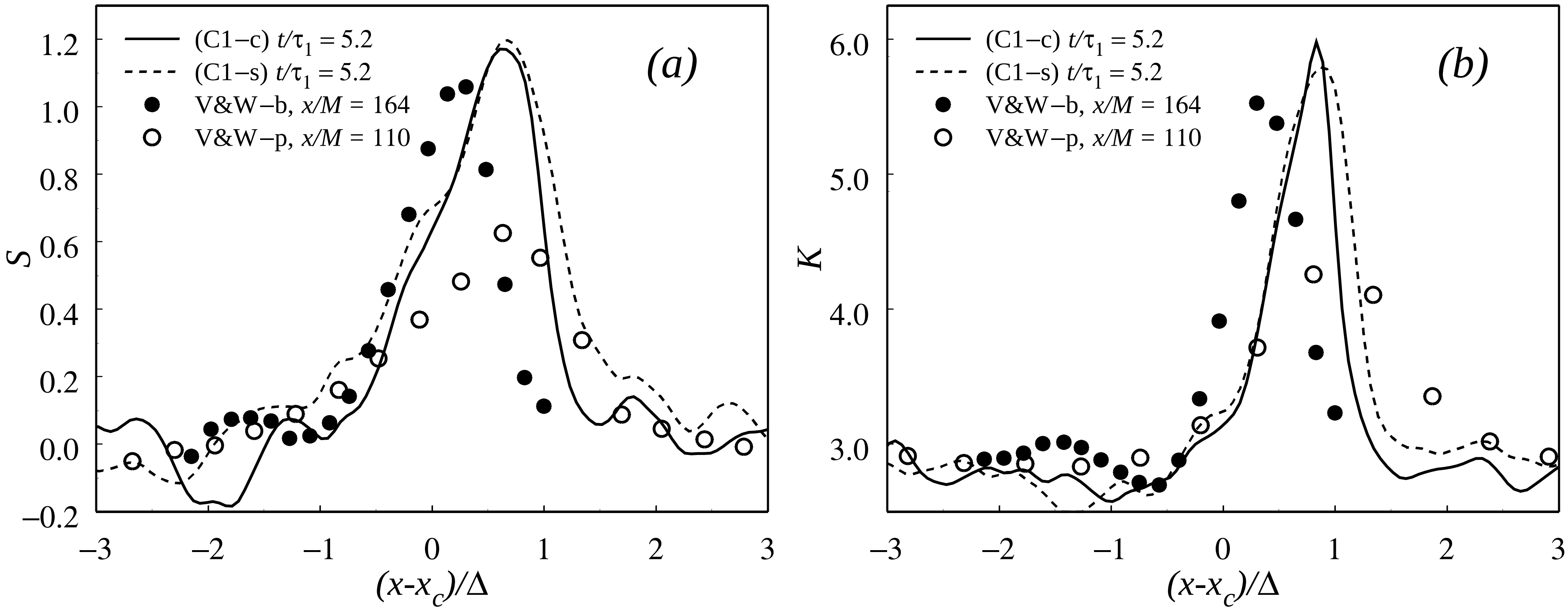
Higher order moments: skewness and kurtosis profiles

$$S = \frac{\overline{u^3}}{\overline{u^2}^{\frac{3}{2}}} \quad K = \frac{\overline{u^4}}{\overline{u^2}^2} \quad \Rightarrow \quad S \approx 0, \quad K \approx 3 \text{ in homogeneous isotropic turb.}$$

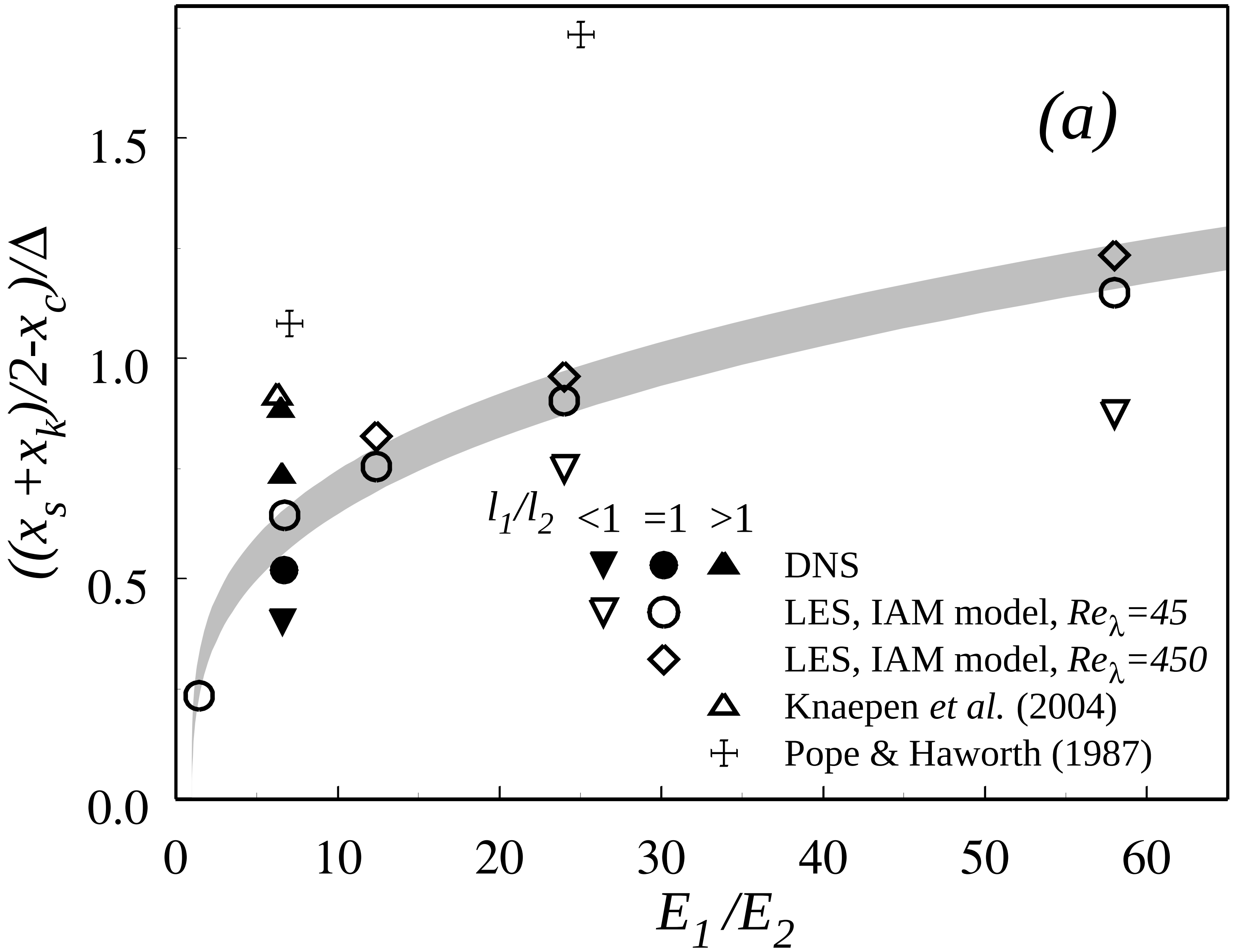
Case A: $\mathcal{E} = 6.7$, $\mathcal{L} = 1$, the two fields have the same integral scale.

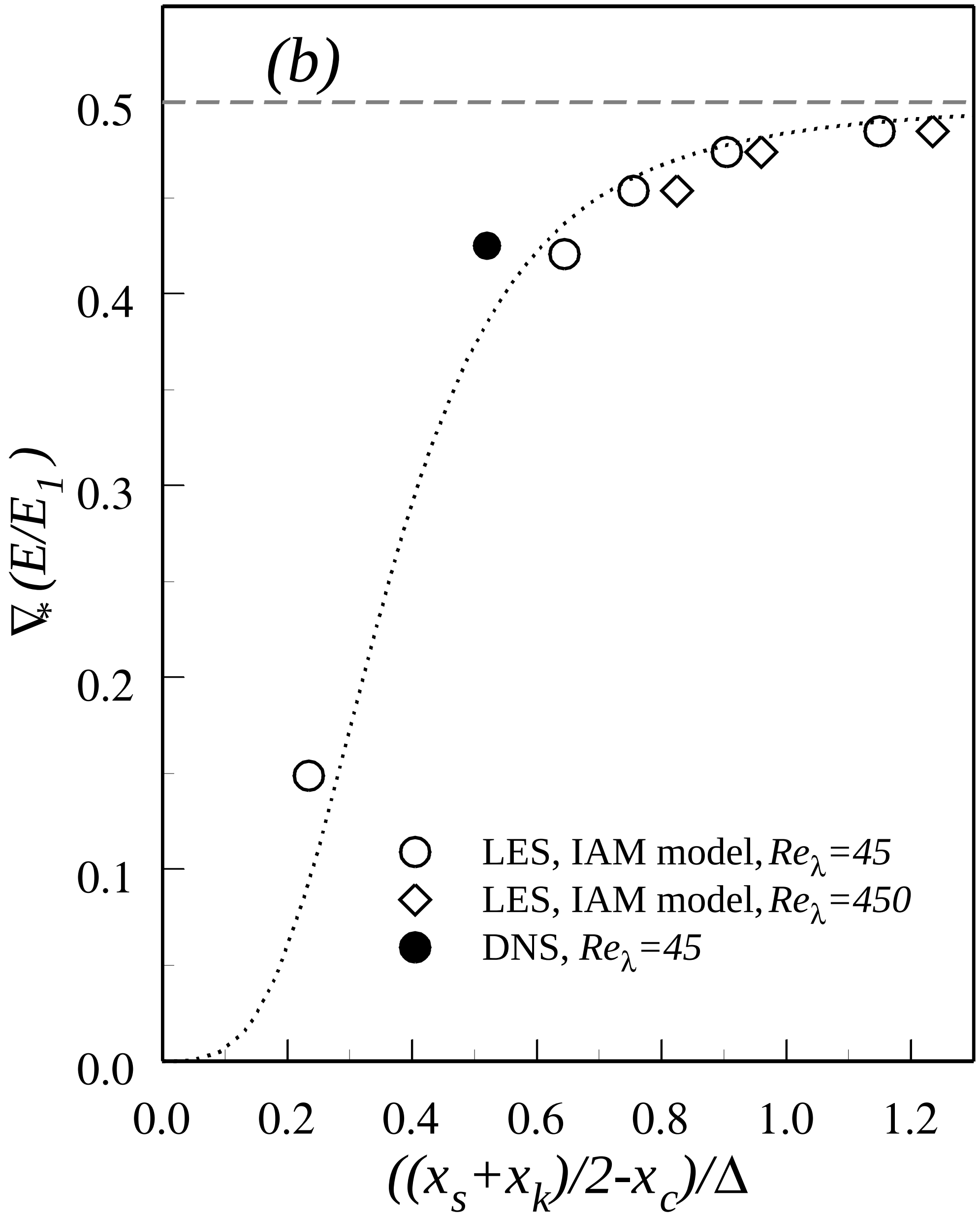


Case C: $\mathcal{E} = 6.5, \mathcal{L} = 1.5$: the gradients of energy and scales have the same sign: larger scale turbulence has more energy



Penetration - position of the maximum of skewness/kurtosis





Penetration with $\mathcal{L} = 1$

Scaling law (energy ratio):

$$\frac{\eta_s + \eta_k}{2} \sim a\left(\frac{E_1}{E_2} - 1\right)^b$$

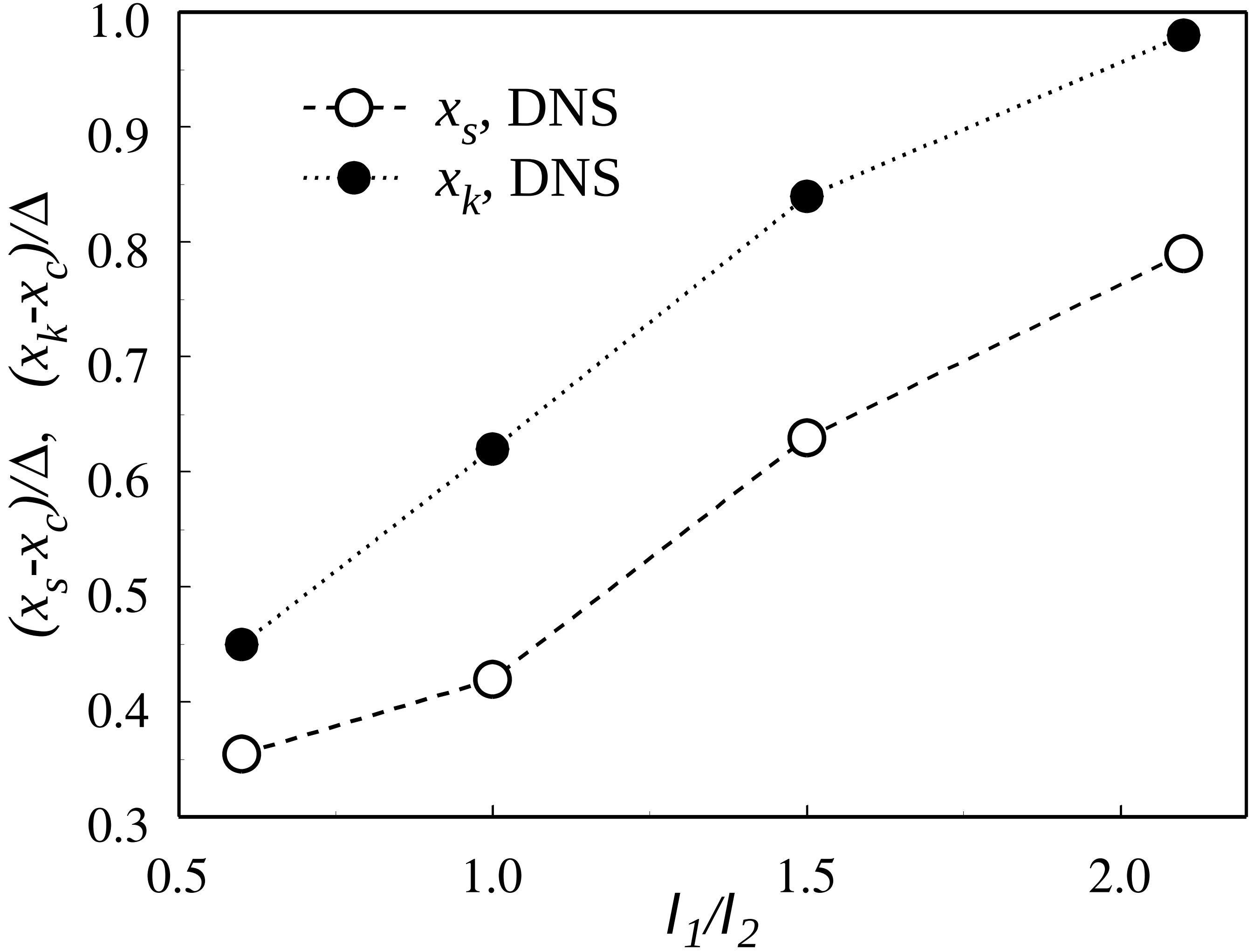
$$a \simeq 0.36, \quad b \simeq 0.298$$

Scaling law (energy gradient):

$$\nabla_*(E/E_1) \simeq (1 - \mathcal{E}^{-1})/2$$

$$\frac{\eta_s + \eta_k}{2} \sim a\left(\frac{2\nabla_*(E/E_1)}{1 - 2\nabla_*(E/E_1)}\right)^b$$

Penetration - position of maximum of skewness/kurtosis, $\mathcal{E} = 6.7$



Part 2: similarity analysis

Properties of the numerical solutions:

- A self-similar decay is always reached
- It is characterized by a strong intermittent penetration, which depends on the two mixing parameters:
  - the turbulent energy gradient
  - the integral scale gradient

This behaviour must be contained in the turbulent motion equations:

- the two-point correlation equation which allows to consider both the macroscale and energy gradient parameters $(B_{ij}(\mathbf{x}, \mathbf{r}, t) = \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x} + \mathbf{r}, t)})$;
- the one-point correlation equation, the limit $\mathbf{r} \to \mathbf{0}$, which allows to consider the effect of the energy gradient only.

Single-point second order correlation equations

To carry out the similarity analysis for $\mathcal{L} = 1$, we consider the second order moment equations for single-point velocity correlations

$$\partial_t \overline{u^2} + \partial_x \overline{u^3} = -2\rho^{-1}\partial_x \overline{pu} + 2\rho^{-1}\overline{p\partial_x u} - 2\varepsilon_u + \nu\partial_x^2 \overline{u^2} \quad (1)$$

$$\partial_t \overline{v_1^2} + \partial_x \overline{v_1^2 u} = 2\rho^{-1}\overline{p\partial_{y_1} v_1} - 2\varepsilon_{v_1} + \nu\partial_x^2 \overline{v_1^2} \quad (2)$$

$$\partial_t \overline{v_2^2} + \partial_x \overline{v_2^2 u} = 2\rho^{-1}\overline{p\partial_{y_2} v_2} - 2\varepsilon_{v_2} + \nu\partial_x^2 \overline{v_2^2} \quad (3)$$

where:
u is the velocity fluctuation in the inhomogeneous direction x,
v_1, v_2 are the velocity fluctuations in the plane (y_1, y_2) normal to x,
ε is the dissipation.

boundary conditions:

outside the mixing, turbulence is homogeneous and isotropic:

- For $x \to -\infty$ (high-energy turbulence):

$$\overline{u^2} = \overline{v_1^2} = \overline{v_2^2} = \frac{2}{3} E_1(t)$$

$$\overline{pu} = \overline{u^3} = \overline{v_1^2 u} = \overline{v_2^2 u} = 0$$

- For $x \to +\infty$ (low-energy turbulence):

$$\overline{u^2} = \overline{v_1^2} = \overline{v_2^2} = \frac{2}{3} E_2(t)$$

$$\overline{pu} = \overline{u^3} = \overline{v_1^2 u} = \overline{v_2^2 u} = 0$$

initial conditions:

$$\overline{u^2} = \overline{v_1^2} = \overline{v_2^2} = \begin{cases} \frac{2}{3} E_1(0) & \text{if} \quad x < 0 \\ \frac{2}{3} E_2(0) & \text{if} \quad x \geq 0 \end{cases} \qquad \overline{pu} = 0$$

Hypothesis and semplifications

- The two homogenous turbulences decay in the same way, thus

$$E_1(t) = A_1(t+t_0)^{-n_1}, \quad E_2(t) = A_2(t+t_0)^{-n_2}$$

  the exponents n_1, n_2 are close each other (numerical experiments, Tordella & Iovieno, 2005). Here, we suppose $n_1 = n_2 = n = 1$, a value which corresponds to $R_\lambda \gg 1$ (Batchelor & Townsend, 1948).

- In the absence of energy production, the pressure-velocity correlation has been shown to be approximately proportional to the convective fluctuation transport (Yoshizawa, 1982, 2002)

$$-\rho^{-1}\overline{pu} = a\,\frac{\overline{u^3} + 2\overline{v_1^2 u}}{2}, \quad a \approx 0.10,$$

- Single-point second order moments are almost isotropic through the mixing:

$$\overline{u^2} \simeq \overline{v_i^2}$$

These semplifications imply that the pressure-velocity correlations can be represented as:

$$-\rho^{-1}\overline{pu} = \alpha\overline{u^3}, \quad \alpha = \frac{3a}{1+2a} \approx 0.25.$$

Thus the problem is reduced to

$$\partial_t\overline{u^2} + (1-2\alpha)\partial_x\overline{u^3} = -2\varepsilon_u + \nu\partial_x^2\overline{u^2}$$

with the boundary and initial conditions previously described.

Similarity hypothesis

The moment distributions are determined by

- the coordinates x, t
- the energies $E_1(t)$, $E_2(t)$
- the scales $\ell_1(t)$, $\ell_2(t)$.

Thus, through dimensional analysis,

$$\begin{aligned}
\overline{u^2} &= E_1 \varphi_{uu}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}) \\
\overline{u^3} &= E_1^{\frac{3}{2}} \varphi_{uuu}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}) \\
\varepsilon_u &= E_1^{\frac{3}{2}} \ell_1^{-1} \varphi_{\varepsilon_u}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}),
\end{aligned}$$

where:

$\eta = x/\Delta(t)$, $\Delta(t)$ is the mixing layer thickness, $R_{\ell_1} = E_1^{\frac{1}{2}}(t)\ell_1(t)/\nu$,

$\vartheta_1 = tE_1^{\frac{1}{2}}(t)/\ell_1(t)$, $\mathcal{E} = E_1(t)/E_2(t)$, $\mathcal{L} = \ell_1(t)/\ell_2(t)$

The high Reynolds number algebraic decay ($n = 1$) implies:

$$
\begin{aligned}
\mathcal{E} &= \text{const} = \frac{E_1(0)}{E_2(0)} \\
\mathcal{L} &= \text{const} = \frac{\ell_1(0)}{\ell_2(0)} \\
\vartheta_1 &= \text{const} = \frac{n}{f(R_{\lambda_1})} \\
R_{\ell_1} &\propto t^{1-n} = \text{const}
\end{aligned}
$$

where $f(R_\lambda) = \dfrac{\varepsilon \ell}{E^{3/2}}$ is constant during decay (see Batchelor (1953), Speziale (1995), Sreenivasan (1998)).

$\Rightarrow \eta$ is the only similarity variable, $\eta = \eta(x,t)$.

⇒ similarity conditions:

By introducing the similarity relations in the equation and by imposing that all the coefficients must be independent from x, t, it is obtained

$$\Delta(t) \propto \ell_1(t)$$

⇒ similarity equation:

$$-\frac{1}{2}\eta\frac{\partial\varphi_{uu}}{\partial\eta} + \frac{1}{f(R_{\lambda_1})}(1-2\alpha)\frac{\partial\varphi_{uuu}}{\partial\eta} + \frac{\nu}{A_1 f(R_{\lambda_1})^2}\frac{\partial^2\varphi_{uu}}{\partial\eta^2} = \\ = \varphi_{uu} - \frac{2}{f(R_{\lambda_1})}\varphi_{\varepsilon_u}$$

with boundary conditions

$$\lim_{\eta\to-\infty}\varphi_{uu}(\eta) = \frac{2}{3}, \quad \lim_{\eta\to+\infty}\varphi_{uu}(\eta) = \frac{2}{3}\mathcal{E}^{-1}, \quad \lim_{\eta\to\pm\infty}\varphi_{uuu}(\eta) = 0$$

$\Rightarrow$ the third-order moment, φ_{uuu}, can be represented as a function of the second order moment, which yields

$$\varphi_{uuu} = \frac{1}{(1-2\alpha)}\left[\frac{f}{2}\int_{-\infty}^{\eta} \eta \frac{\partial \varphi_{uu}}{\partial \eta} \mathrm{d}\eta + \frac{\nu}{A_1 f}\frac{\partial \varphi_{uu}}{\partial \eta}\right]$$

$$S = \frac{\varphi_{uu}^{-\frac{3}{2}}}{(1-2\alpha)}\left[\frac{f}{2}\int_{-\infty}^{\eta} \eta \frac{\partial \varphi_{uu}}{\partial \eta} \mathrm{d}\eta + \frac{\nu}{A_1 f}\frac{\partial \varphi_{uu}}{\partial \eta}\right]$$

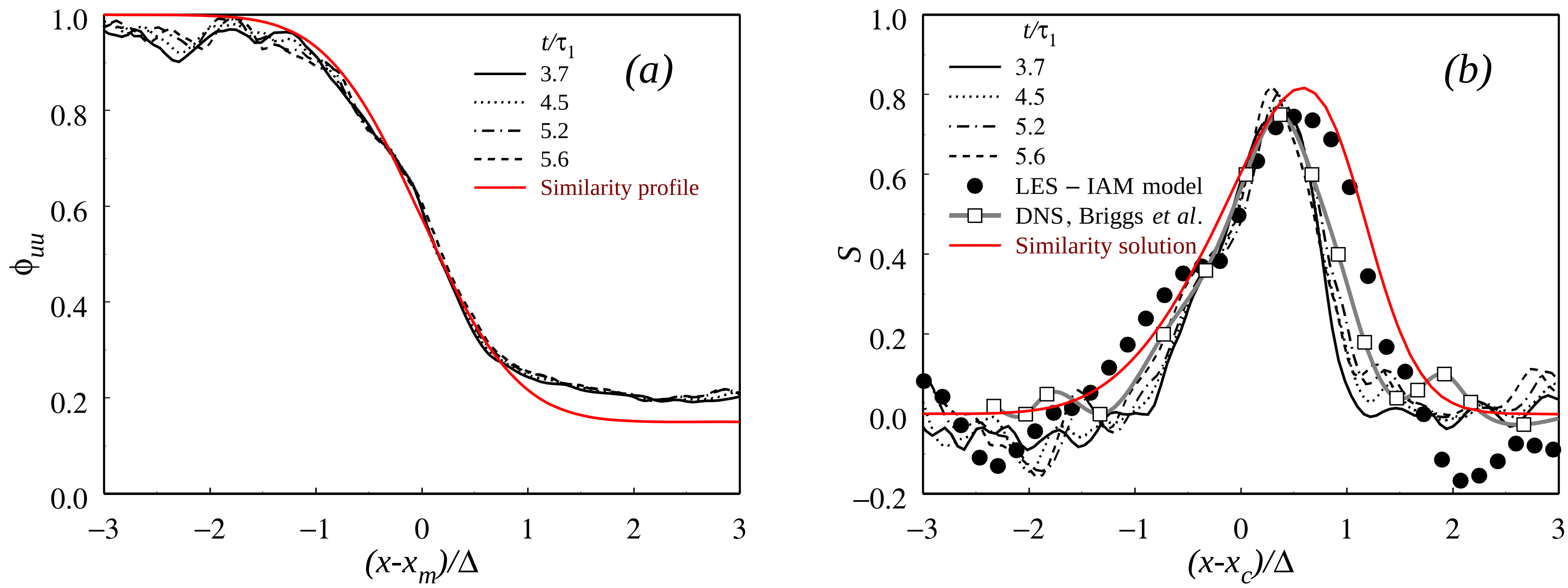
With regard to the second-order moments, the numerical experiments suggest the fit (see also Veeravalli & Wahrhaft, *JFM 1989*)

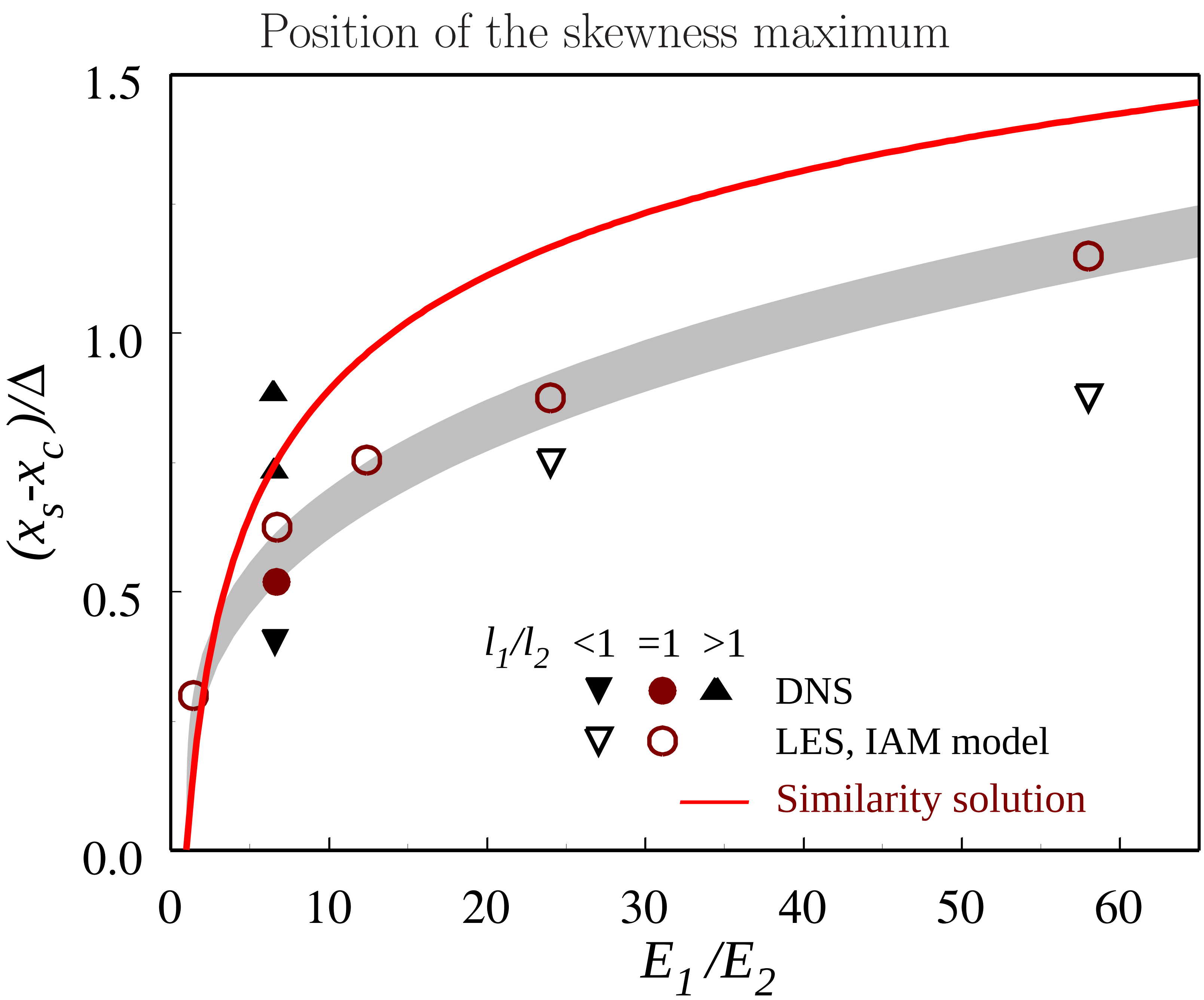
$$\frac{3}{2}\varphi_{uu} = \frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta),$$

This allows to compute the velocity skewness by analitical integration

$$S = \frac{1-\mathcal{E}^{-1}}{\sqrt{\pi}}\mathrm{e}^{-\eta^2}\left[\frac{f}{4(1-2\alpha)}\left(1-\frac{4\nu}{A_1 f^2}\right)\right] \times \left[\frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta)\right]^{-\frac{3}{2}}$$

Normalized energy and skewness distributions; $\mathcal{E} = 6.7$ and $\mathcal{L} = 1$.





Position of the skewness maximum
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
$(x_s - x_c)/\Delta$
E_1/E_2
l_1/l_2 <1 =1 >1
DNS
LES, IAM model
Similarity solution

Conclusions (... up to now)

The intermediate asymptotics of the turbulence diffusion in the absence of production of turbulent kinetic energy is considered.

- An intermediate similarity stage of decay always exists.
- When the energy ratio $\mathcal{E}$ is far from unity, the mixing is very intermittent.
- when $\mathcal{L} = 1$, the intermittency increases with the energy ratio $\mathcal{E}$ with a scaling exponent that is almost equal to 0.29.
- intermittency smoothly varies when passing through $\mathcal{L} = 1$:
  it increases when $\mathcal{L} > 1$ (*concordant* gradient of energy and scale),
  it is reduced when $\mathcal{L} < 1$ (opposite gradient of energy and scale)
- the self-similar decay of the shearless mixing is consistent with the similarity solution of the single-point correlation equation.

. . . work in progress

- Consider both scale and energy gradient pareameters by means of the two-point correlation equation
- Small scales. . . → velocity derivative skewness and structure functions
- Reynolds number effect
- Computational accuracy: influence of the domain dimensions

(. . . the end)

Appendix: Numerical discretization...

Incompressible Navier-Stokes equations:

$$\begin{aligned}\partial_t u_i + \partial_j (u_i u_j) &= -\partial_i p + \frac{1}{Re}\nabla u_i \\ -\nabla^2 p &= \partial_i \partial_j (u_i u_j)\end{aligned}$$

Cubic domain ($2\pi \times 2\pi \times 2\pi$) with periodic boundary conditions:

$$u_i(\mathbf{x} + 2\pi \mathbf{e}^{(j)}) = u_i(\mathbf{x}) \quad \forall i, j$$

Fourier-Galerkin approximation (see Canuto et al., 1988):

$$u_i^N(\mathbf{x}, t) = \sum_{k_1,k_2,k_3=-N/2}^{N/2} \hat{u}_{i,\mathbf{k}}^N(t) \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}, \quad p^N(\mathbf{x}, t) = \sum_{k_1,k_2,k_3=-N/2}^{N/2} \hat{p}_{\mathbf{k}}^N(t) \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$$

Semi-discrete equations:

$$\begin{aligned}\partial_t \hat{u}_i^N &= -\mathrm{i}k_j \widehat{(u_i u_j)} - \mathrm{i}k_i \hat{p}^N - \frac{k^2}{Re}\hat{u}_i^N \\ -k^2 \hat{p}^N &= -k_i k_j \widehat{(u_i u_j)}, \quad k^2 = \mid \mathbf{k} \mid^2\end{aligned}$$

Convective terms $\widehat{(u_i u_j)}$ are evaluated with pseudo-spectral method.
Time integration with low storage Runge-Kutta 4 (Jameson at al. *AIAA J.* 1981)

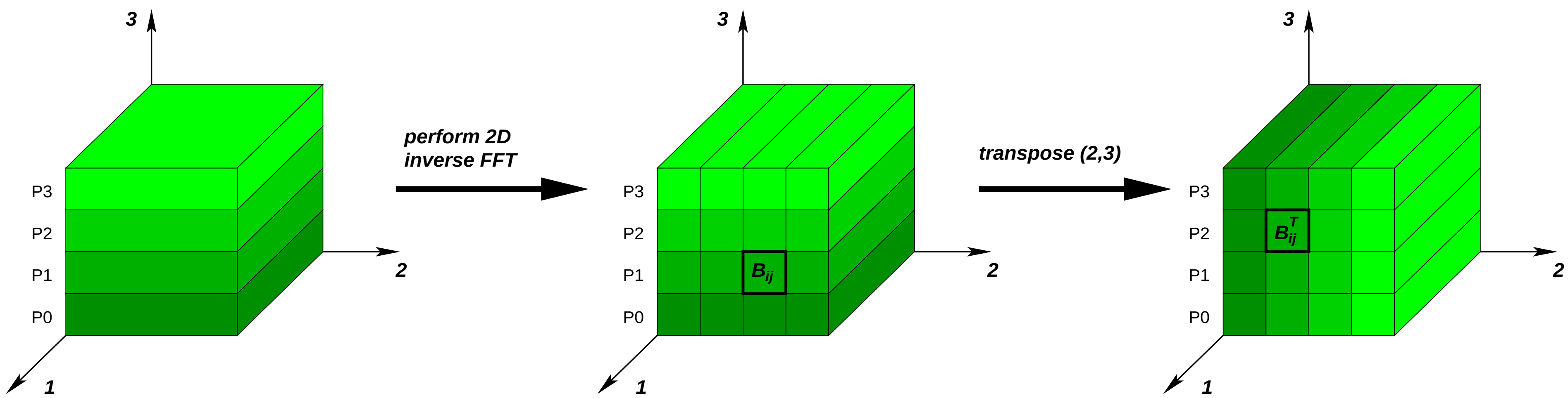
. . . Parallel code

The code uses real-to-real FFT and stores Fourier coefficients in hermitian form (see Iovieno-Cavazzoni-Tordella, *Comp.Phys.Comm.* 2001)
Most operations are local in the wavenumber space with the exception of the pseudo-spectral computation of convective terms $(\widehat{u_i u_j})$:

Basic method (aliasing error)

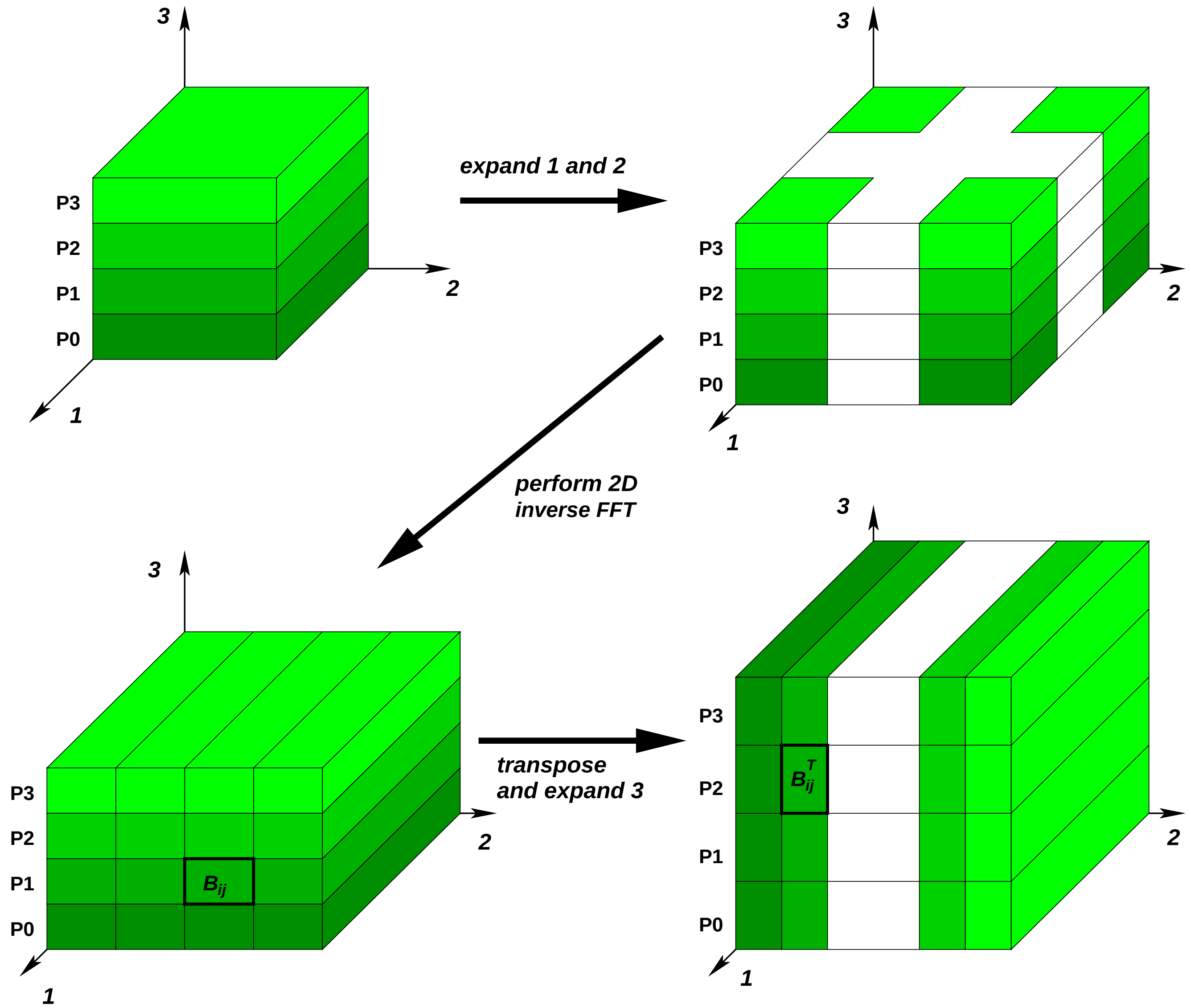
- inverse FFT of $\hat{u}_i^N$ and $\hat{u}_j^N$
- product in the physical space
- FFT of the product

Scheme for parallel FFT/inverse FFT



$\Rightarrow$ To remove the aliasing error data must be expansed from N to $M = \frac{3}{2}N$ points in alla directions (see Canuto et al., 1988).

. . . dealiased pseudo-spectral computation of products



(Scheme for parallel inverse FFT)

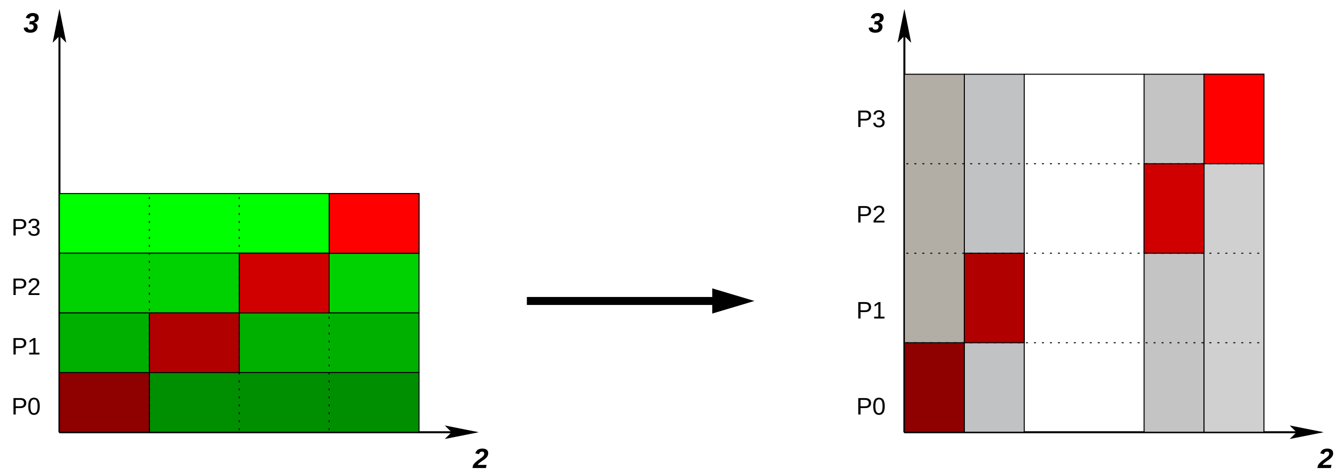
Data transposition (inverse FFT)

- ***Preliminary cicle***: each processor transposes and collocates the "diagonal block":

$$g(j_1, j_2 + I(I_{i_{rank}}), j_3) \leftarrow f(j_1, j_3 + i_{rank} M_{loc}, j_2)$$

where

$$I(i) = \begin{cases} N_{proc} i & if i < N_{proc} \\ I(i) = M - N_{proc} i & \text{otherwise} \end{cases}$$



N, $M = 3N/2$ are the number of points,
N_{proc} is the number of processors $N_{loc} = N/N_{proc}$, $M_{loc} = M/N_{proc}$

- ***Main loop:***

for j from 0 to $N_{proc} - 1$,

– each processor defines the destination and source for communication:

$$idest = irank + j,\ isource = irank - j$$

– each processor creates and transposes the block to be sent:

$$B^T(j_1, j_2, j_3) \leftarrow f(j_1, j_3 + i_{dest} N_{loc}, j_2)$$

– Communication occurs by means of a call to `MPI_send_recv_replace`

– Each processor allocates the received block in the new position:

$$g(j_1, j_2 + I(I_{source}), j_3) \leftarrow B^T(j_1, j_2, j_3).$$

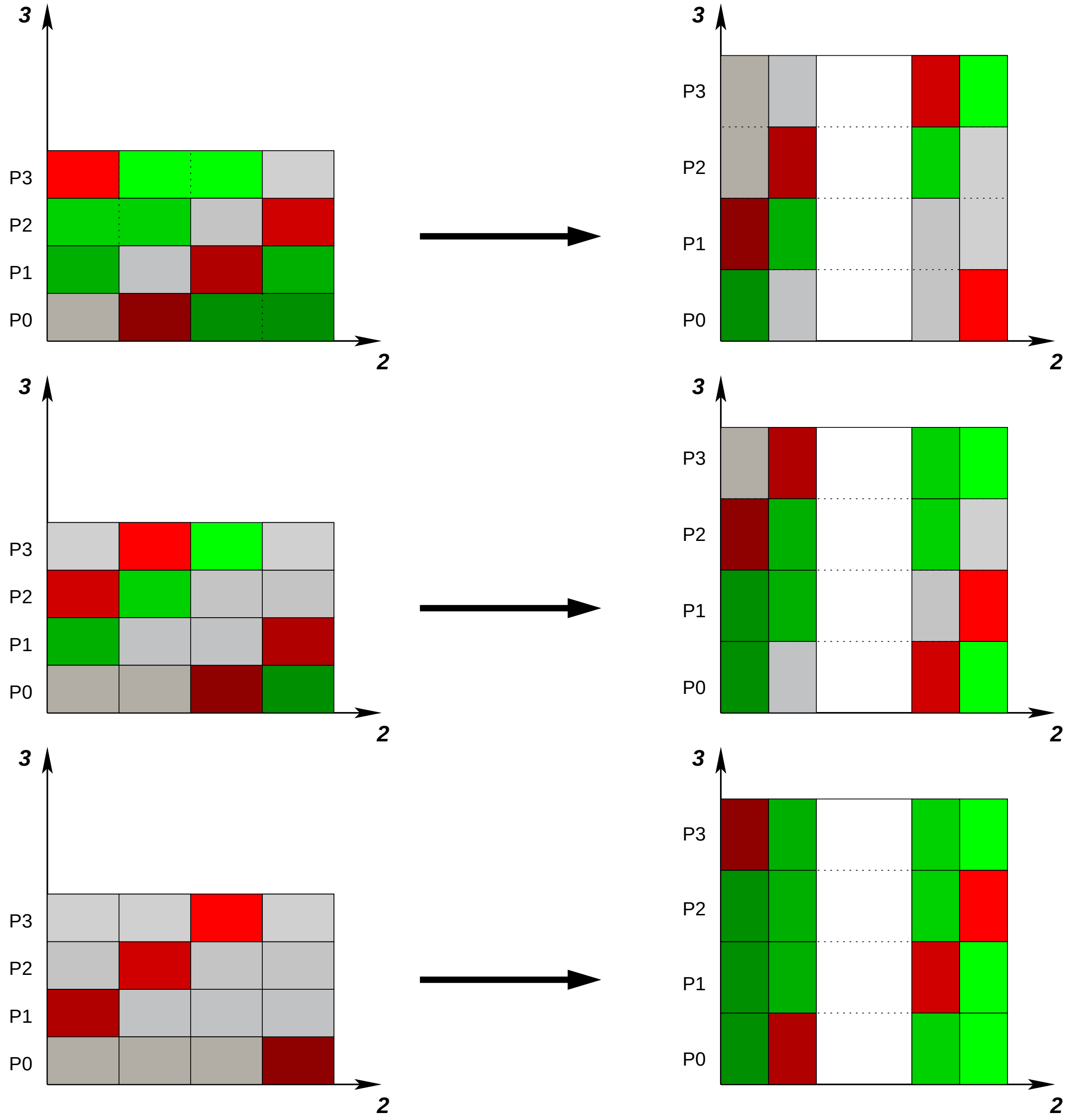
end of the loop.

... example with 4 processors in next slide →

$j = 1$

$j = 2$

$j = 3$



Note: Red blocks are transferred during each step of the loop