The small-scale localization in the large-eddy simulation of turbulent compressible jets

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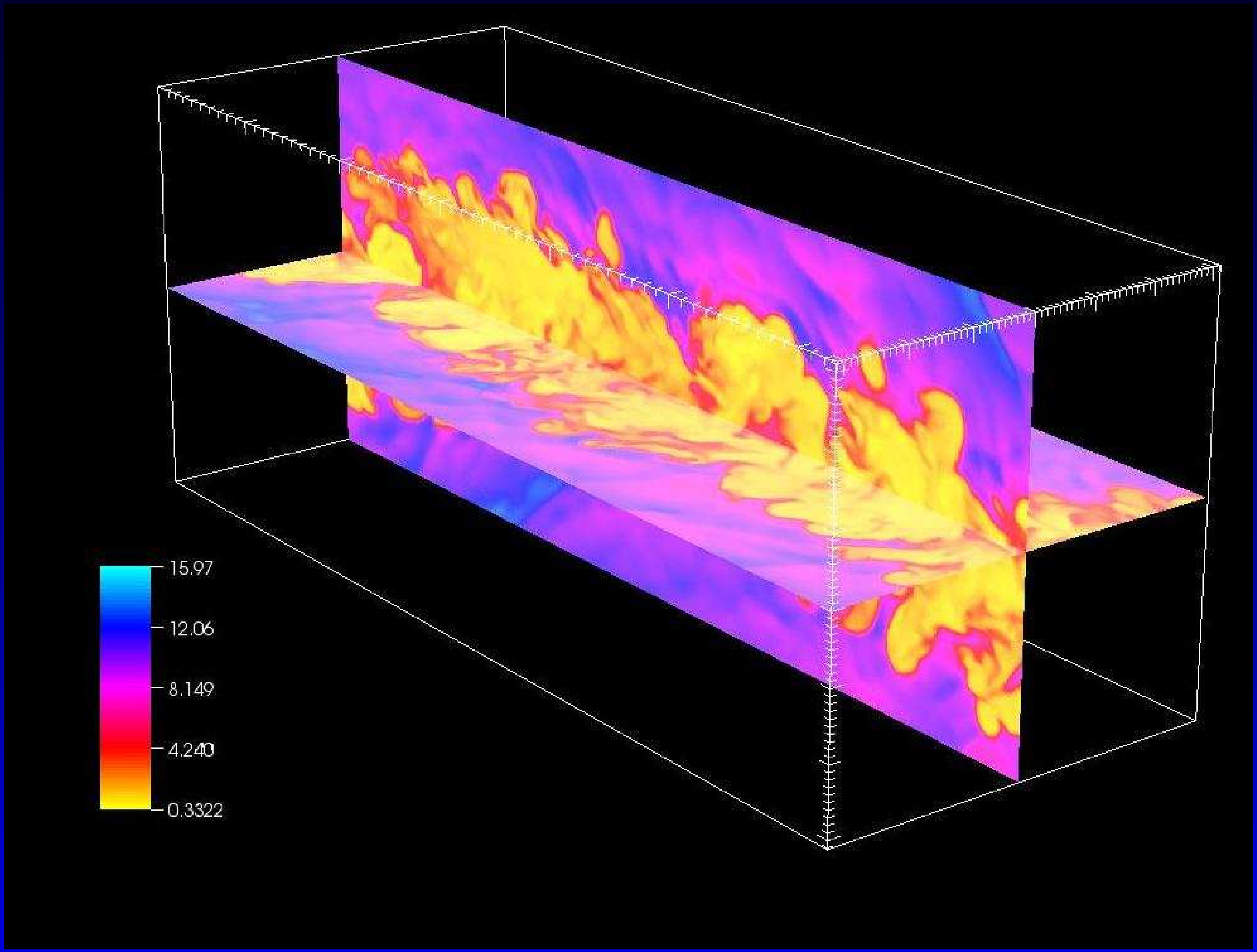
Context



logarithmic density contours, light jet with initial M=5 and density ratio 0.1

*tracer movie**

Context

Hypersonic jets:

- Very high Reynolds numbers,
  e.g.: astrophysical jets have $Re > 10^{10} \div 10^{13}$
  and Ma up to $50 \div 100$
  (Ferrari, *Ann.Rev.A.A.* 1998)
- only the large scales only may be simulated
  $\Rightarrow$ explicit LES modelling is needed.
- Subgrid scales are not present in all regions
  $\Rightarrow$ selective application of LES fluxes

Selective LES

Selective Structure Function model by Lesieur (1996).
It is based on:

$$f(\langle \boldsymbol{\omega} \rangle_\delta) = \frac{\langle \boldsymbol{\omega} \rangle_\delta \cdot \langle \langle \boldsymbol{\omega} \rangle \rangle_{2\delta}}{\mid \langle \boldsymbol{\omega} \rangle_\delta \mid \mid \langle \langle \boldsymbol{\omega} \rangle \rangle_{2\delta} \mid} \in [-1, 1]$$

when f is close to 1 $\Rightarrow \approx$ 2D, no subgrid terms
when f is far to 1 $\Rightarrow$ subgrid scales are present
Problems:
• only the disalignement of vorticity vector on scale δ is used.
• it is not easy to define a threshold, which in turn seems to depend on the resolution.

Some ideas...

When filter cutoff is within the inertial subrange, the smallest resolved scales ($\sim \delta$):

- are highly three-dimensional
- vortex stretching is active and responsible of the energy transfer to smaller (unresolved) scales

$\longrightarrow$ the stretching and the enstrophy at the smallest resolved scale can be used to create a criterion for the localization regions with under-resolved turbulence.

Small scale localization criterion

We consider the following functional (Tordella *et al.*, CPC 2007):

$$f(\langle \mathbf{u} \rangle, \langle \boldsymbol{\omega} \rangle) = \frac{|\,(\langle \boldsymbol{\omega} \rangle - \boldsymbol{\Omega}) \cdot \nabla(\langle \mathbf{u} \rangle - \mathbf{U})\,|}{|\,\langle \boldsymbol{\omega} \rangle - \boldsymbol{\Omega}\,|^2}$$

where $\mathbf{U}$ and $\boldsymbol{\Omega}$ are the mean velocity and vorticity.

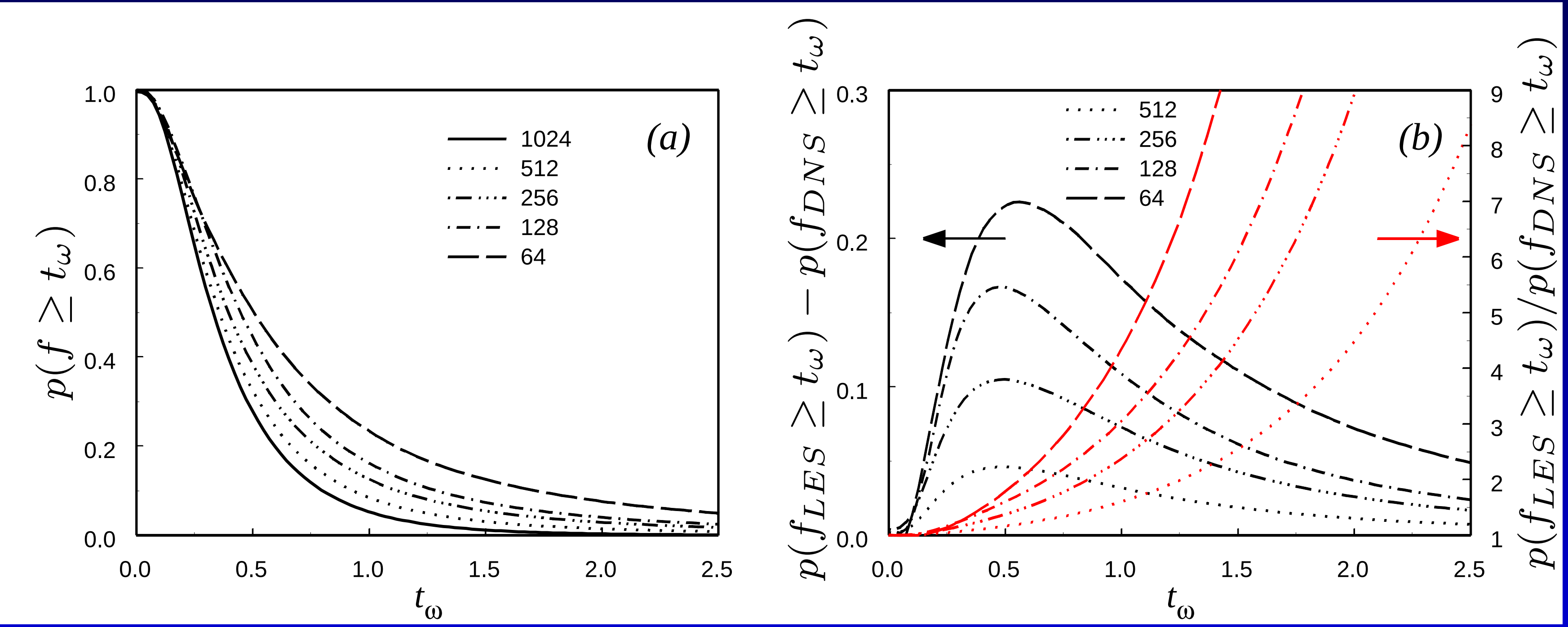
Main features:

- information from only one filtering level – with filter scale Δ – is used.
- f is always positive, ranging from 0 when there is no flow to high values when there is high stretching in relation to enstrophy

A priori **test**

Homogeneous and isotropic turbulence (DNS, $Re_\lambda = 280$, Toschi et.al.2003):

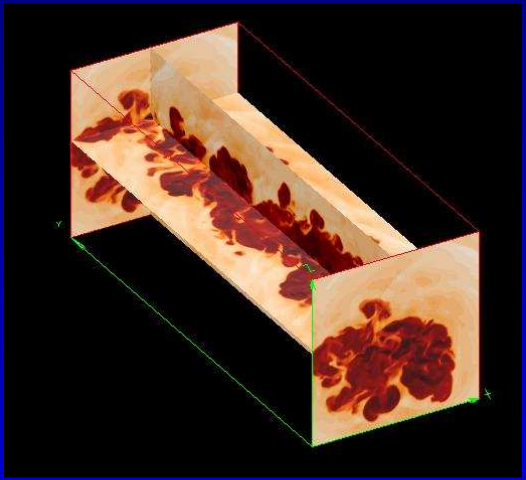
P.d.f. of f at different resolutions.



Compressible supersonic jet

Physical problem

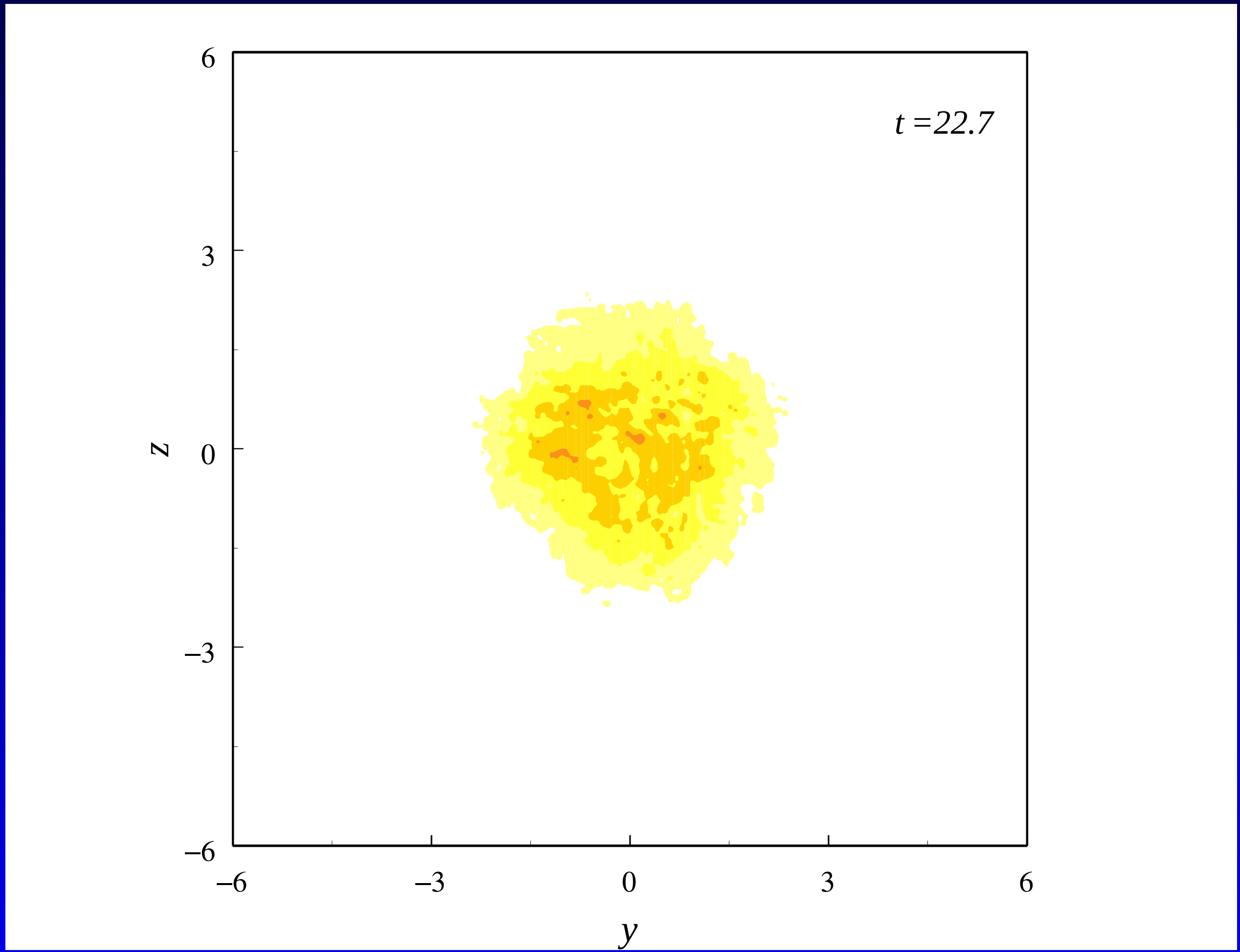
- time decaying jet
- initially uniform jet plus unstable perturbations:
  - Mach number $Ma = 5$
  - density ratio 0.1 (light jet)



Test on previous simulations

$P(f \geq t_\omega),\ t_\omega = 0.4,$

256^3, no selective LES (Micono *et al.* APJ 2000)



Compressible jet simulation

We integrate the following equations: (Favre averaged variables)

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i\right) &= 0 \\
\frac{\partial(\rho u_k)}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i u_k + p\delta_{ik}\right) &= H\left(f_{\mathrm{LES}} - t_\omega\right)\frac{\partial \tau_{ik}^{SGS}}{\partial x_i} \\
\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_i}\left[(E+p)u_i\right] &= H(f_{\mathrm{LES}} - t_\omega)\frac{\partial q_i^{SGS}}{\partial x_i}
\end{aligned}$$

$H(\cdot)$ is the Heaviside step function.

$E = \frac{p}{\gamma - 1} + \rho\frac{|\boldsymbol{u}|^2}{2}$ is the total energy density.

τ_{ik}^{SGS} and q_i^{SGS} are the subgrid flows of momentum and energy

Compressible jet simulation

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$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i\right) = 0$$

$$\frac{\partial (\rho u_k)}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i u_k + p\delta_{ik}\right) = H\left(f_{\mathrm{LES}} - t_\omega\right)\frac{\partial \tau_{ik}^{SGS}}{\partial x_i}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_i}\left[(E+p)u_i\right] = H(f_{\mathrm{LES}} - t_\omega)\frac{\partial q_i^{SGS}}{\partial x_i}$$

$$\tau_{ij}^{SGS} = 2\rho\nu_t\left[S_{ij} - \frac{1}{3}\left(\nabla\cdot\boldsymbol{u}\right)\delta_{ij}\right], \quad q_i^{SGS} = \rho\frac{\nu_t}{Pr_t}\frac{\partial}{\partial x_i}\left(\frac{E+p}{\rho}\right)$$

$\nu_t = \left[C_S\left(\Delta x\Delta y\Delta z\right)^{1/3}\right]^2 |S|$, $C_S = 0.1$, $|S| = \sqrt{2S_{ij}S_{ij}}$, $Pr_t = 1$

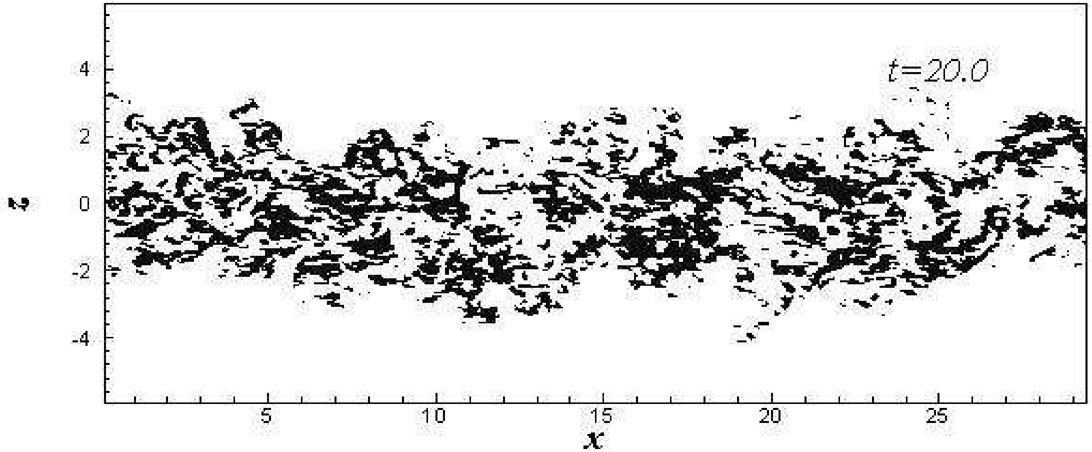
Numerical method

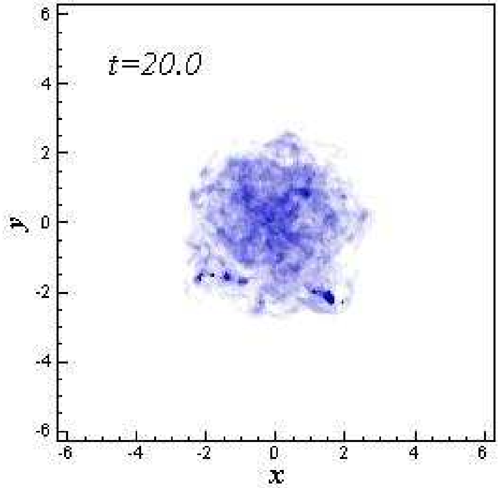
- Discretization (*PLUTO*-code, Mignone *et al.* APJ 2007*)
  - 3^{rd} order PPM finite volume discretization
  - 3^{rd} order explicit Runge-Kutta time integration
- Domain and b.c.:
  - 3D cartesian domain, $4\pi R_0 \times 10\pi R_0 \times 4\pi R_0$ (R_0 is the initial radius of the jet)
  - periodic b.c. (longitudinal direction)
  - non reflecting b.c. (normal directions)

* `http://plutocode.to.astro.it`

Localization of subgrid scales

Regions where $f \geq t_\omega$, $t_\omega = 0.4$, $P(f \geq t_\omega)$

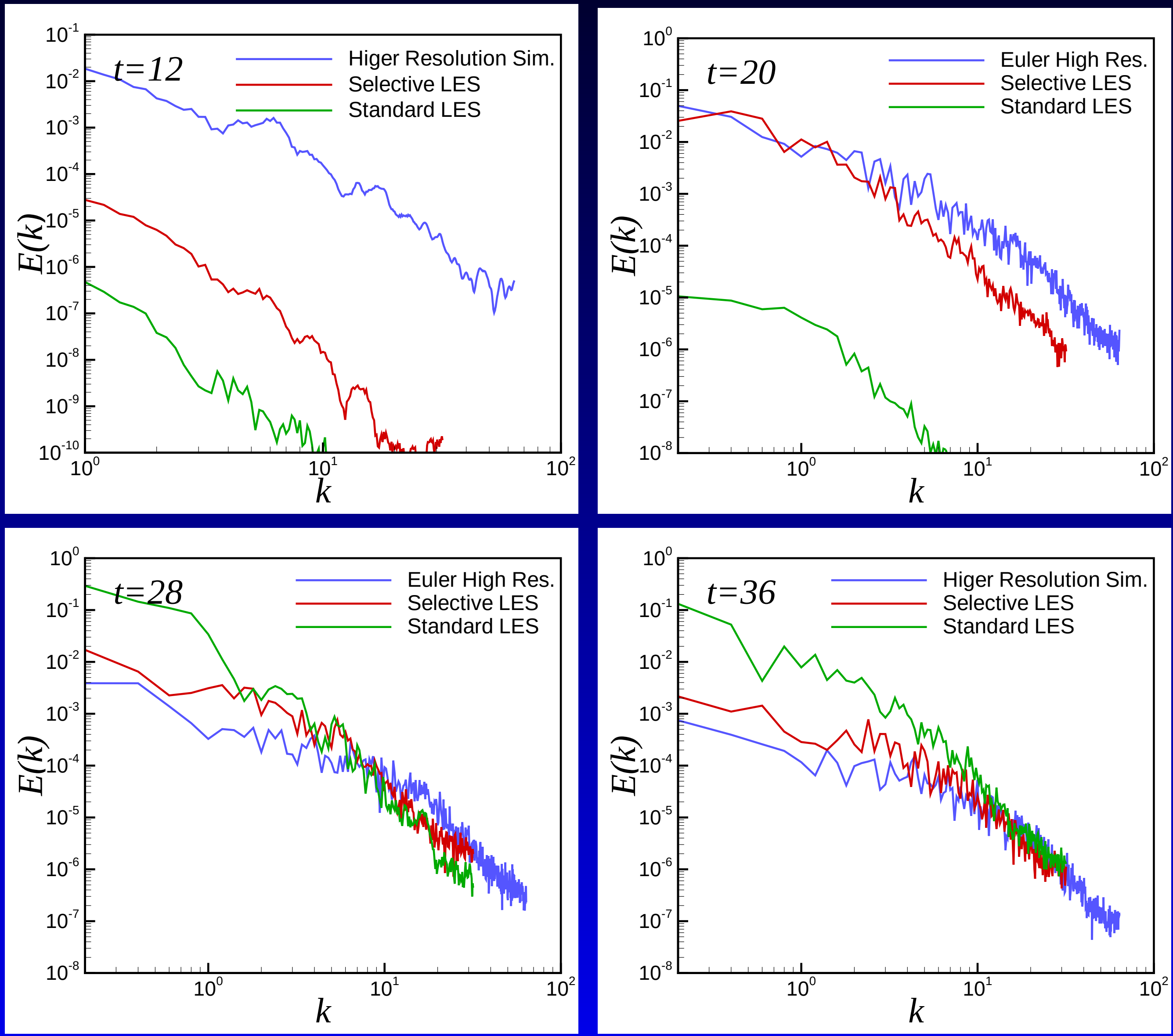




movie f

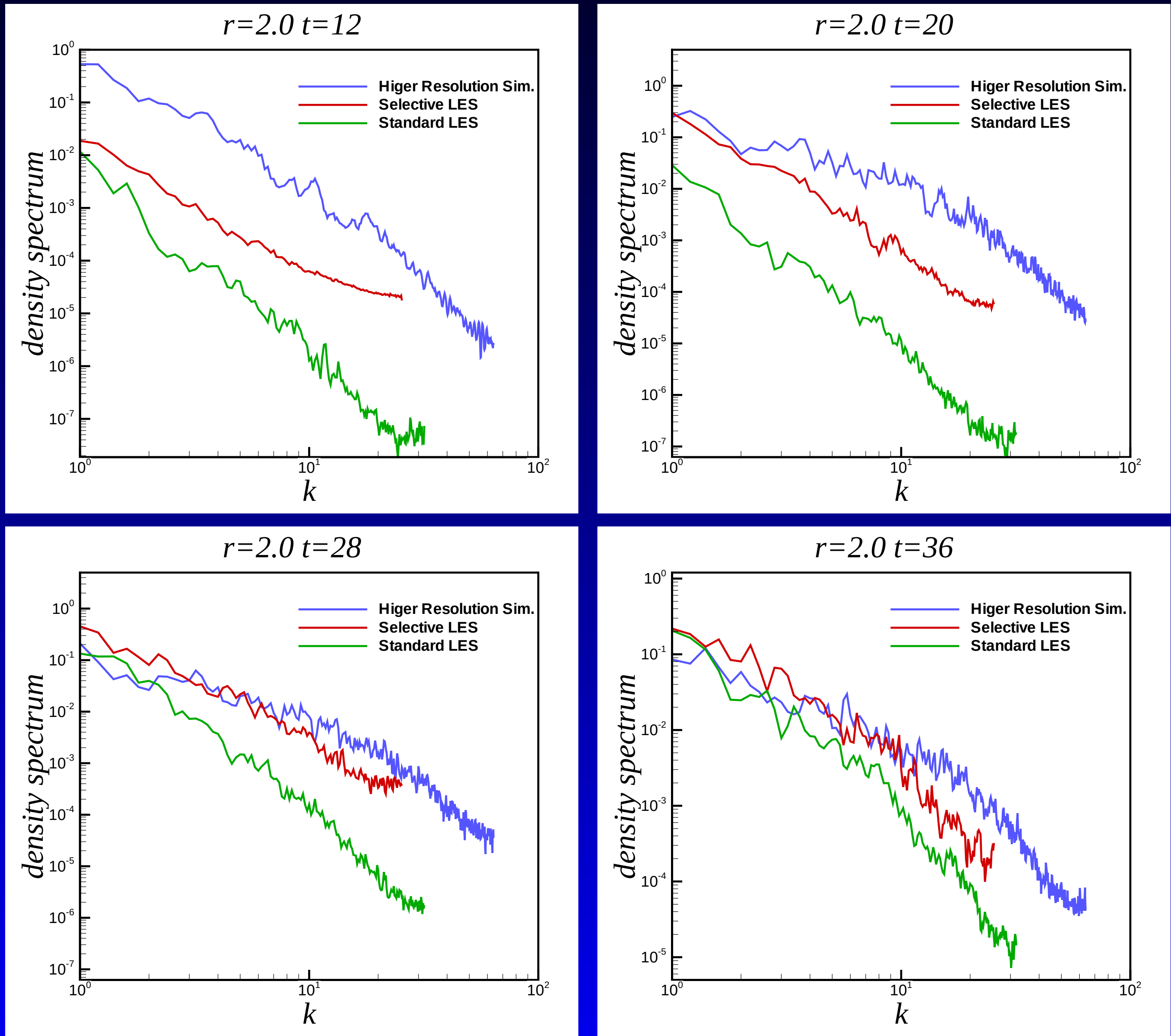
movie pdf(f)

1D energy spectra, $r = 2$



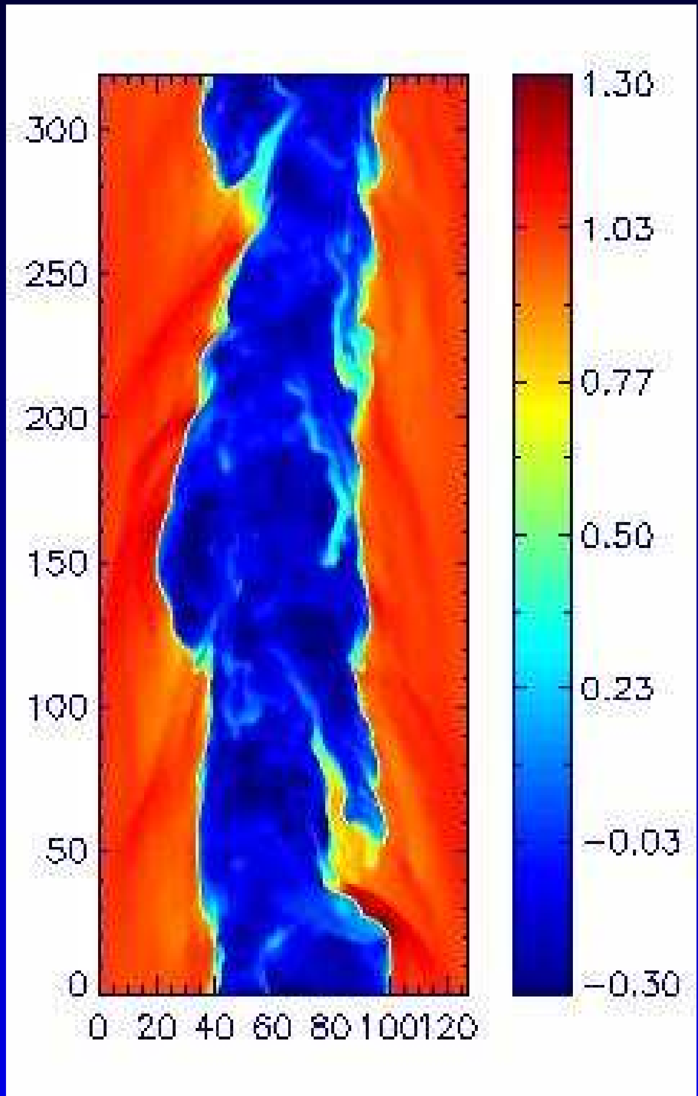
LES $128^2 \times 320$, Euler HR $256^2 \times 640$

1D Density spectrum

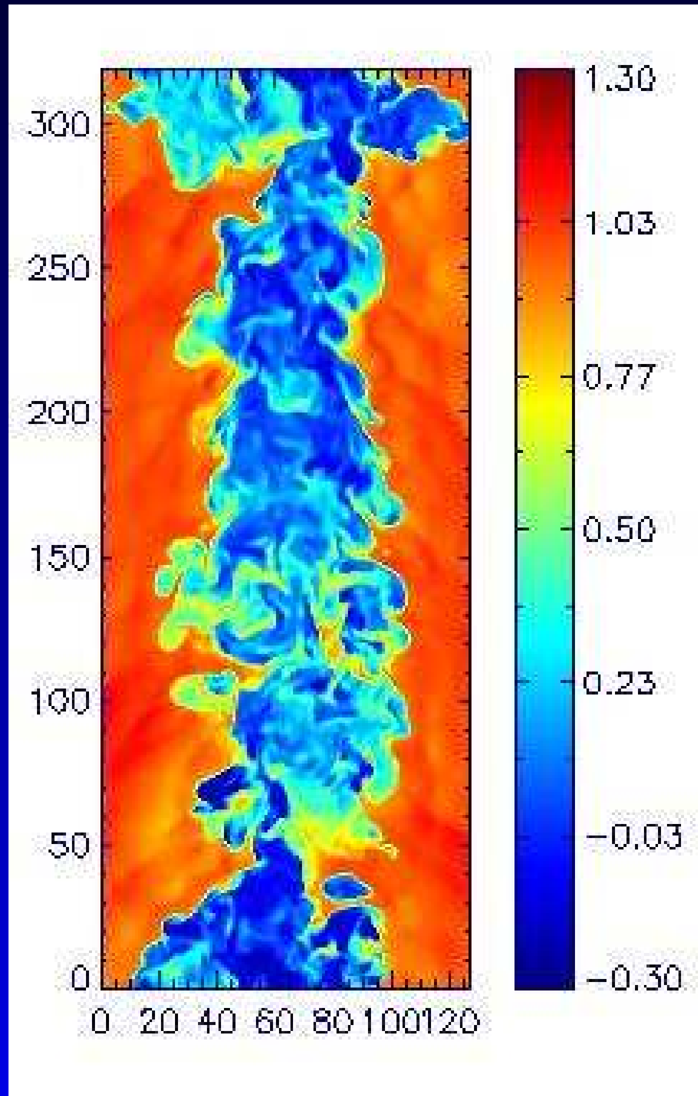


Density, $t = 20$

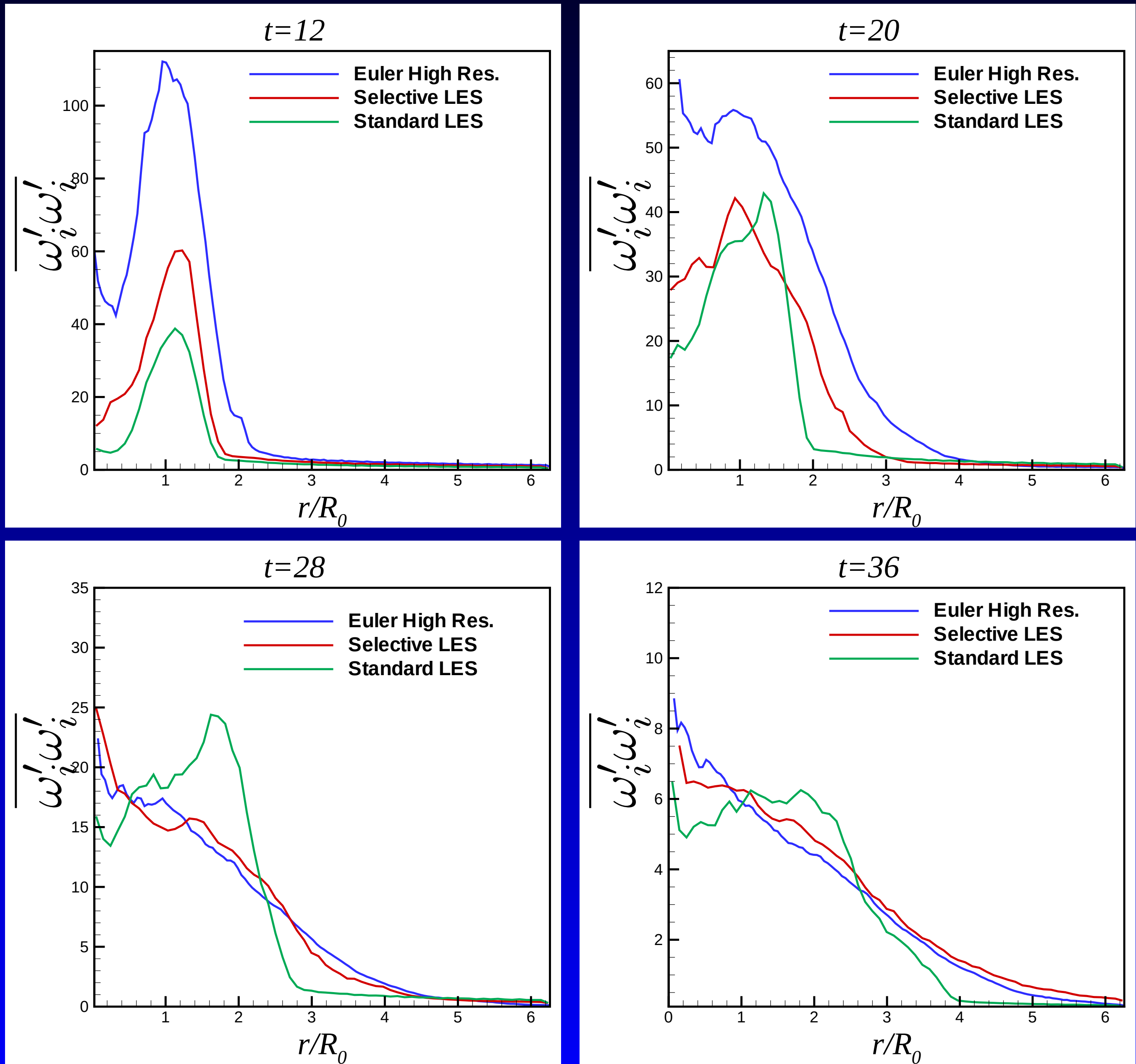
Standard LES



Selective LES



Enstrophy



t=12
t=20
t=28
t=36
Euler High Res.
Selective LES
Standard LES
$\overline{\omega_i'\omega_i'}$
r/R_0

Conclusions

A criterion for small-scale localization has been built on the normalized stretching of the fluctuations

- it is local in space and based on one level of filtered variables
- it is independent of the subgrid scale model
- it is numerically stable
- it shows the capability to locate the highly intermittent zones in a compressible jet
- post processing and further analysis on progress