Diffusion of scalars across a turbulent energy gradient

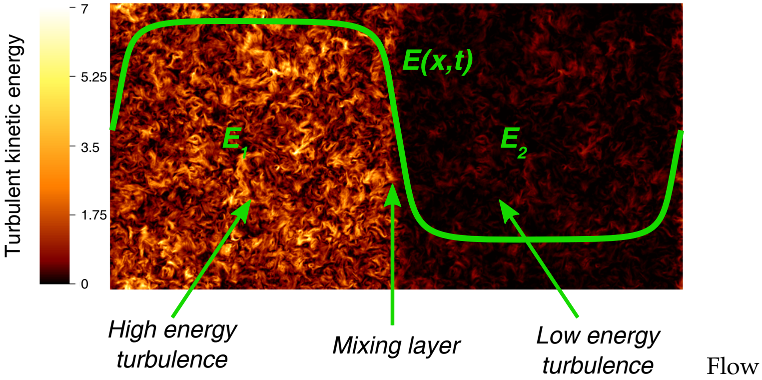
M. Iovieno, L. Ducasse, D. Tordella,
F. De Santi, S. Di Savino[1] and J. Riley[2]

[1]Politecnico di Torino, Dipartimento di Ingegneria Aeronautica e Spaziale
[2]Mechanical Engineering Department, University of Washington, WA

Turbulence Mixing and Beyond, Trieste, August 2011
COST Meeting, Warsaw, September 2011

Turbulent shearless mixing

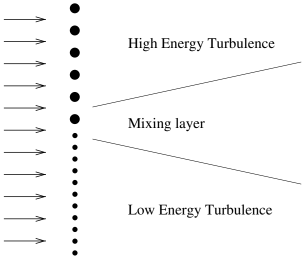
General flow configuration:



Parameters: Reynolds number, Energy Ratio E_1/E_2, Scale ratio ℓ_1/ℓ_2

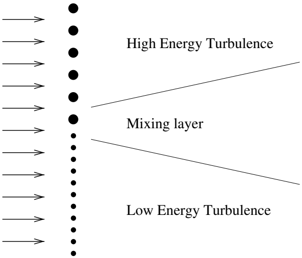
movie

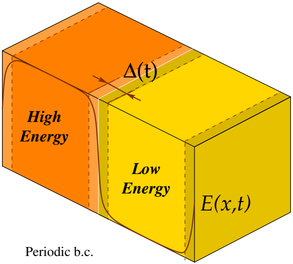
State of the art



- Grid turbulence experiments:
  - Gilbert *JFM* 1980
  - Veeravalli-Warhaft *JFM* 1989

State of the art





- Grid turbulence experiments:
  - Gilbert *JFM* 1980
  - Veeravalli-Warhaft *JFM* 1989

- Numerical experiments:
  - Briggs *et al. JFM* 1996
  - Knaepen *et al. JFM* 2004
  - Tordella-Iovieno *JFM* 2006
  - Iovieno-Tordella-Bailey *PRE* 2008
  - Kang-Meneveau *Phys.Fluids* 2008
  - Tordella-Iovieno *Phys.Rev.Lett.* (under revision)

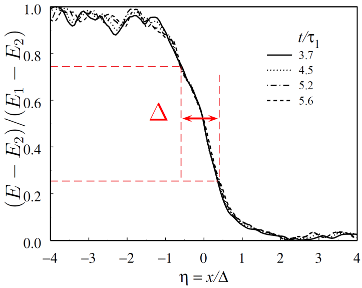
Main features of mixing layers

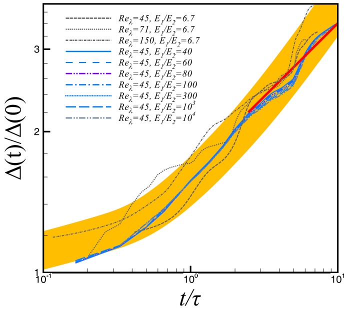
Shearless mixing layers shows the following properties:

- no gradient of mean velocity, thus no kinetic energy production
- the mixing is generated by the inhomogeneity in the turbulent kinetic energy and integral scale
- the mixing layer becomes very intermittent at both large and small scales [Tordella-Iovieno *Phys.Rev.Lett.* 2011]
- the presence of a gradient of energy is a sufficient condition for the onset of intermittency [Tordella and Iovieno *JFM* 2006; Tordella et al. *Phys. Rev.* 2008]
- 2D and 3D mixings: different asymptotic layer thickness growth exponent

3D mixing: Self-similarity

$E_1/E_2 = 6.7, \ell_1 = \ell_2$

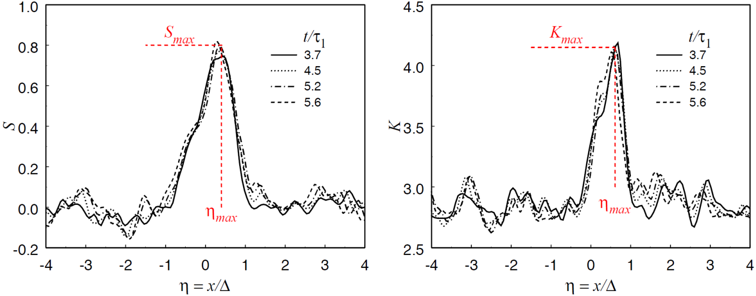




$\Delta(t)$ is the conventional mixing layer thickness, $\Delta(t) \sim t^{0.46}$

Large scale intermittency

$S = \overline{u^3}/\overline{u^2}^{3/2}$ $K = \overline{u^4}/\overline{u^2}^2$

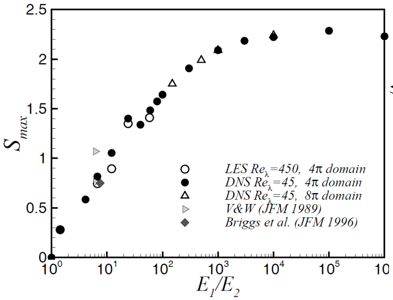


u = velocity component in the mixing direction
S_{max}, K_{max} = maximum of Skewness and Kurtosis in the mixing layer
η_{max} = normalized position of the maximum in the mixing layer

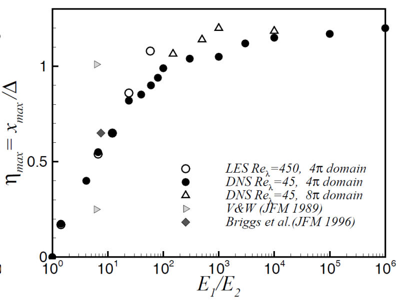
(Figures: sample data from simulations with $E_1/E_2 = 6.7$, $\ell_1 = \ell_1$, $Re_\lambda = 45$)

Intermittency vs. Energy ratio

Skewness

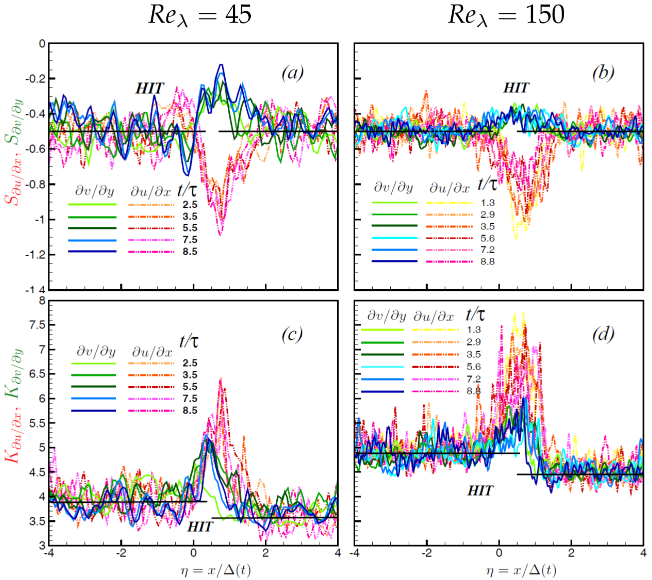


Penetration



We define the penetration as the position of the maximum of the skewness normalized over the mixing layer thickness: $\eta = \frac{x_s(t)}{\Delta(t)}$

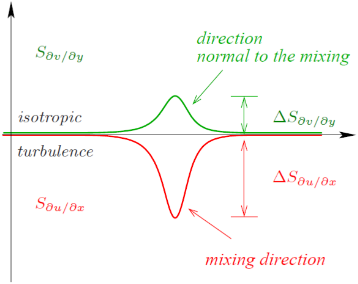
Velocity derivative

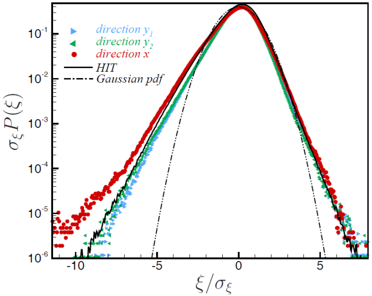


Phys.Rev.Lett., 2011 (under revision)

Velocity derivative skewness

General behaviour

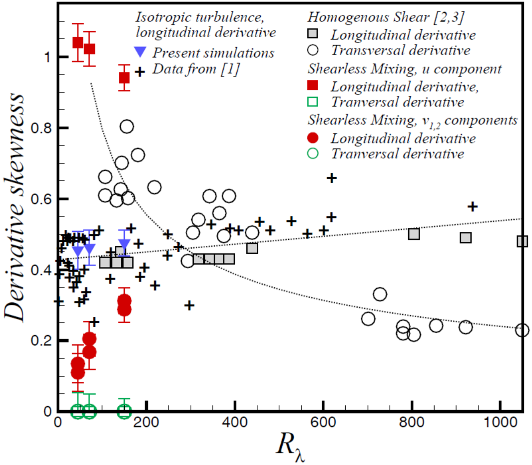




$\xi = \partial u_i/\partial x_i$, $i = x, y_1$ and y_2
$(Re = 150, t/\tau = 3.5)$

Increase of fluid filaments compression in the energy gradient direction, reduction of fluid filaments compression in the other directions

Small scale anisotropy



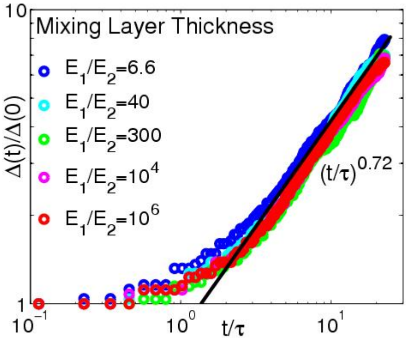
(1) Sreenivasan-Antonia *Ann.Rev.Fluid Mech* 1997

(2,3) Warhaft-Shen *Phys.Fluids* 2000 and 2002.

Shear flows: large transiversal skewness

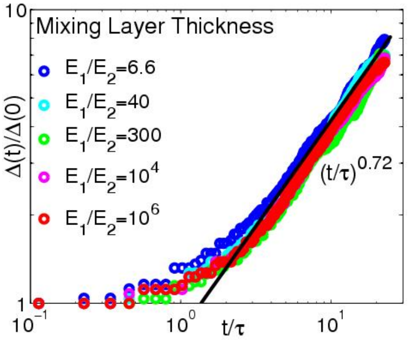
Shearless mixings: strong differentiation of the longitudinal skewness

2D - 3D Comparison



$$2D : \frac{\Delta(t)}{\Delta(0)} \propto \left(\frac{t}{\tau}\right)^{0.72} \qquad 3D : \frac{\Delta(t)}{\Delta(0)} \propto \left(\frac{t}{\tau}\right)^{0.43}$$

2D - 3D Comparison



$2D: \frac{\Delta(t)}{\Delta(0)} \propto \left(\frac{t}{\tau}\right)^{0.72}$ $3D: \frac{\Delta(t)}{\Delta(0)} \propto \left(\frac{t}{\tau}\right)^{0.43}$

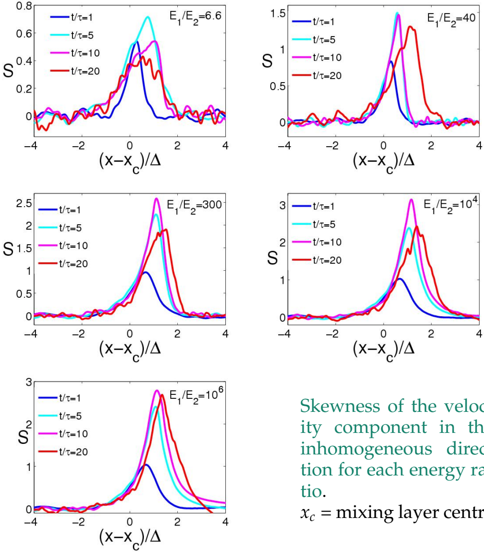
2D turbulent diffusion is infinitely grater than 3D diffusion: by defining a diffusion velocity as $v_{\mathcal{D}} = \mathrm{d}x_s/\mathrm{d}t = \eta \mathrm{d}\Delta/\mathrm{d}t$ we have

$v_{\mathcal{D}} = \propto t^{-0.28}$ $v_{\mathcal{D}} = \propto t^{-0.57}$

movie

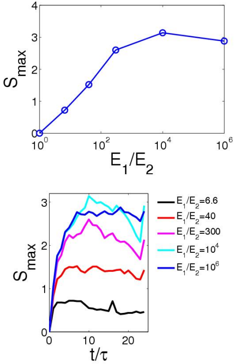
Skewness

2D mixing



Skewness of the velocity component in the inhomogeneous direction for each energy ratio.

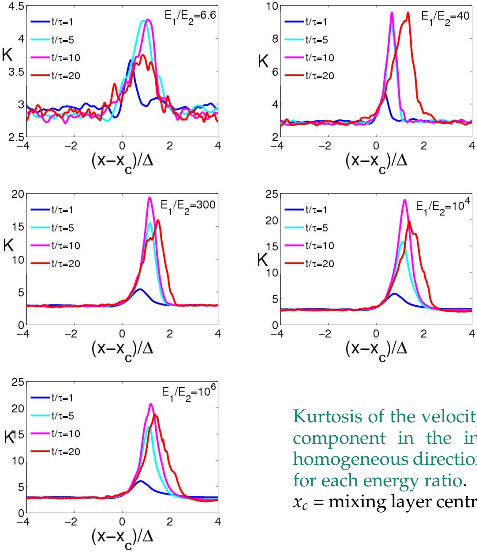
x_c = mixing layer centre



Maximum of the Skewness as a function of the energy ratio and of the time

Kurtosis

2D mixing



Kurtosis of the velocity component in the inhomogeneous direction for each energy ratio.
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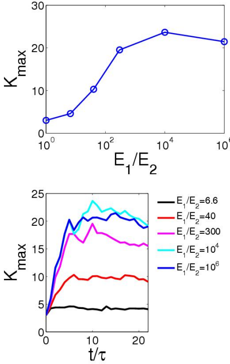
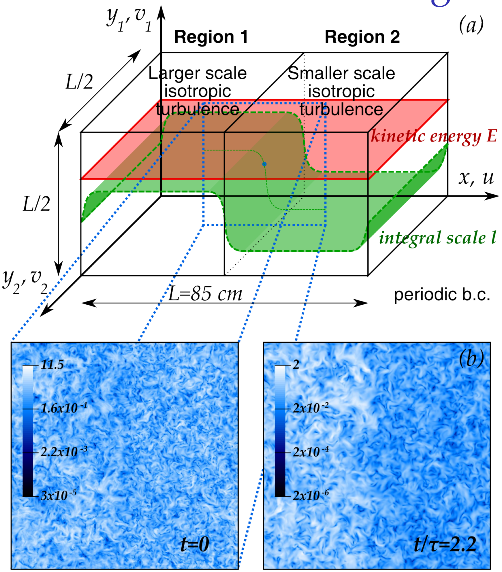


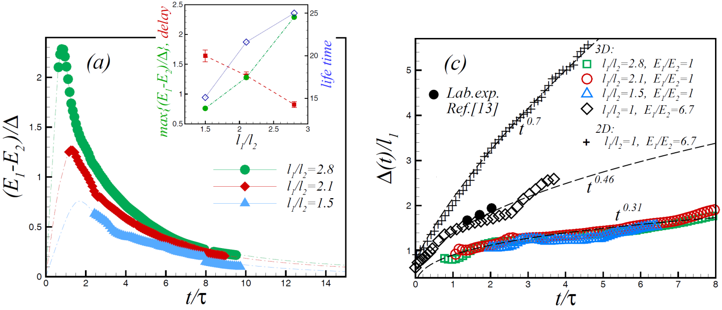
Maximum of the kurtosis as a function of the energy ratio and of the time

Uniform kinetic energy, inhomogeneous scale



Physica D, 2011 (in press).

Energy gradient generation

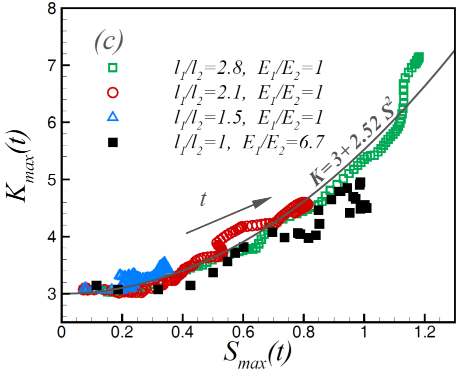


Different decay exponents of the homogenous regions

$\Rightarrow$ generation of an *energy gradient*

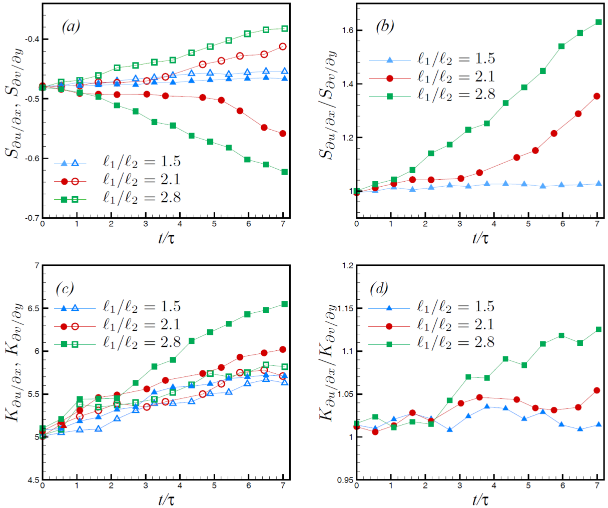
Velocity moments

Skewness vs. Kurtosis during the decay



Velocity derivative

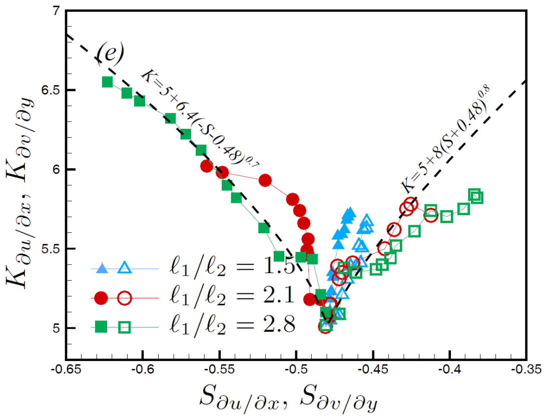
Longitudinal derivative Skewness and Kurtosis



Left (a-c): Filled symbols $\partial u/\partial x$, empty symbols $\partial v/\partial y$

Velocity derivative

Longitudinal skewness vs. longitudinal kurtosis



Filled symbols $\partial u/\partial x$, empty symbols $\partial v/\partial y$

Conclusions

Uniform energy - inhomogeneous scale

- different scales generate different decays and then an energy gradient concurrent to the scale gradient
- the transient lifetime of the kinetic energy gradient is almost proportional to the initial scale ratio
- intemittency in the interaction layer grows as the flow decays
- anisotropy and intermittency are, with a certain lag, spread also to small scales
- small scale anisotropy: strong differentiation of the longitudinal skewness but no transversal skewness

Passive scalar

Basic phenomenology

- A passive scalar is a contaminant present in so low concentration that it has no dynamical effect on the fluid motion.
- Turbulence transports the scalar by making particles follow chaotic trajectories and disperses the scalar concentration if exists scalar concentration gradient.
- Fluctuations reach the smaller scales.

Passive scalar

Basic phenomenology

- at large scales:
  - the mean concentration, variance and flux are strongly influenced by the boundary conditions and scalar injection
- at small scales:
  - scalar differences are not gaussian,
  - intermittency observed at inertial range scales as well as at the dissipation scales, with ramp/cliff structures

see, e.g.:
Warhaft *Ann.Rev.F.M.* 2000,
Shraiman and Siggia, *Nature* 2000,
Gotoh, *Phys.Fl.* 2006, 2007.

Passive scalar transport

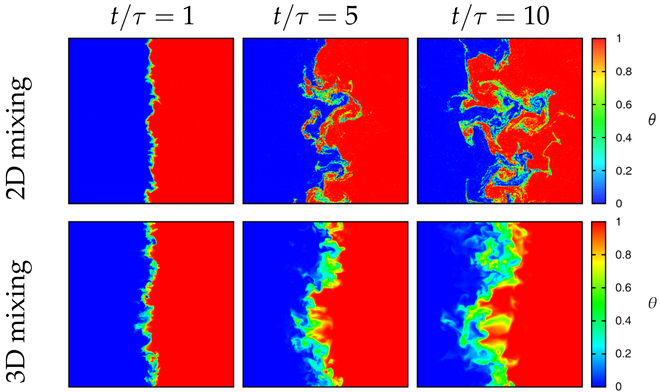
We solve the passive scalar advection-diffusion equation

$$\frac{\partial \vartheta}{\partial t} + u_j \frac{\partial \vartheta}{\partial x_j} = \frac{(-1)^{n+1}}{Re\, Sc} \nabla^{2n} \vartheta$$

for the shearless mixing configuration with $E_1/E_2 = 6.6$, $\ell_1 = \ell_2$.

DNS simulations have been performed at $Re_\lambda = 150$ in 3D turbulence ($600^2 \times 1200$ grid points, $n = 1$) and $Re_\lambda = 60$ in 2D turbulence (1024^2 grid points, $n = 2$).
Schmidt number $Sc = 1$

Passive scalar concentration



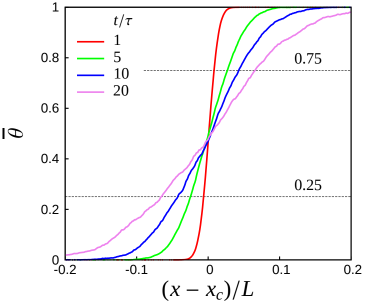
2D movie

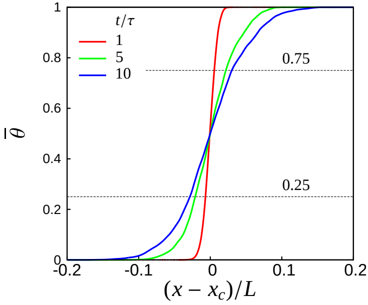
3D movie

Mean Scalar Diffusion

2D Mixing

3D Mixing

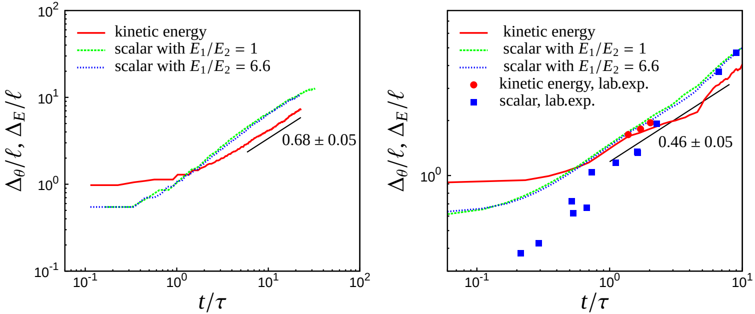




Energy ratio $E_1/E_2 = 6.6$

Scalar mixing layer thickness

2D Mixing 3D Mixing

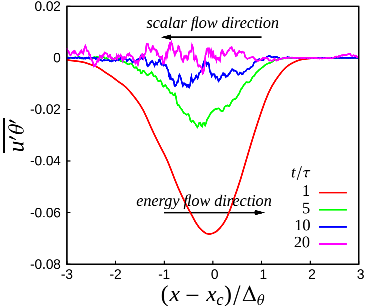


Scalar layer thickness: $\Delta_\vartheta = x_{(\vartheta=0.75)} - x_{(\vartheta=0.25)}$

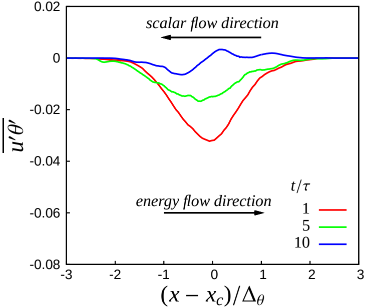
3D mixing: $\Delta_\vartheta \sim t^{0.46}$, 2D mixing: $\Delta_\vartheta \sim t^{0.68}$

Scalar flux

2D Mixing



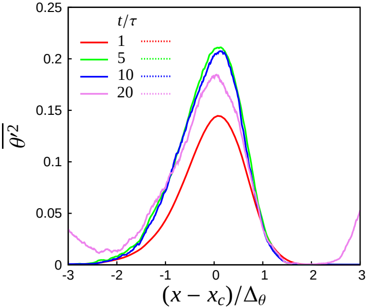
3D Mixing



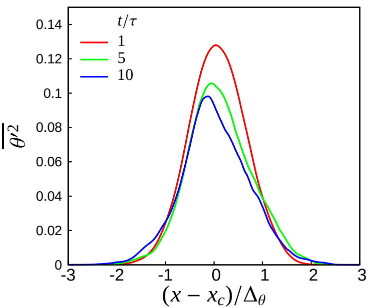
$$\overline{u'\vartheta'} \sim 1/\Delta_\vartheta(t)$$

Scalar variance

2D Mixing



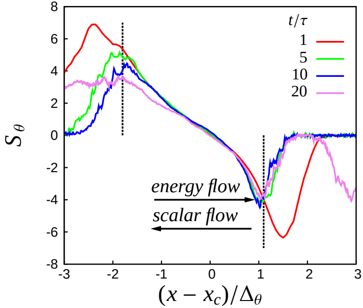
3D Mixing



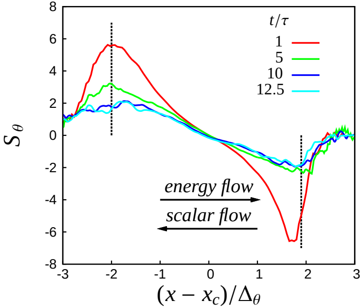
Self-similar distribution, peak shifted toward the high kinetic energy region

Scalar skewness

2D Mixing



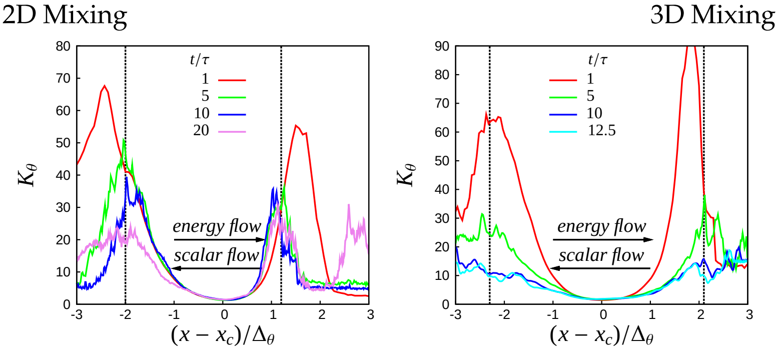
3D Mixing



Strong non-gaussian statistic at the mixing layer border

2D: intermittency penetrates more in the direction opposite to the energy gradient.

Scalar kurtosis



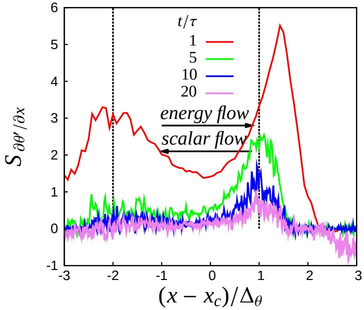
2D: higher asymmetry of the peaks.

Intermittency gradually reduces as the mixing procedes

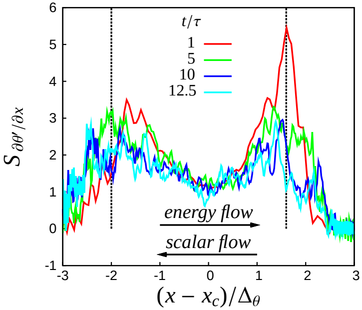
Small scale intermittency

Scalar derivative skewness

2D Mixing



3D Mixing

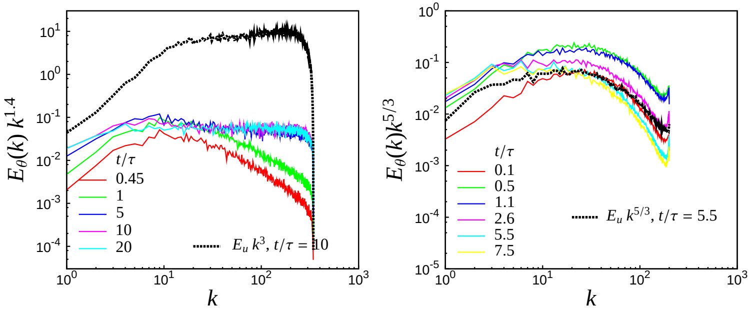


2D: higher asymmetry of the peaks.

Intermittency decay faster in 2D

Spectra in the mixing layer

2D Mixing 3D Mixing



Compensated scalar and velocity one-dimensional spectra in the same position

Passive scalar - Main remarks

- Growth rate:
  2D flow : $(\Delta_\vartheta \sim \Delta_E \sim t^{0.68})$, 3D flow : $(\Delta_\vartheta \sim \Delta_E \sim t^{0.46})$.
- Self-similar profiles of first and second order moments. The scalar flow is about two times larger in 2D than in 3D. The scalar variance in the center of the mixing layer is 50% higher in 2D case.
- Large intermittency and non-gaussian behaviour on both sides of the mixing, even where the scalar flux is small.
- Larger asymmetry in moment distributions in 2D mixing.
- intermittency involves also the small scales
- inertial range spectra exponent:
  scalar: 3D $\sim -5/3$, 2D ~ -1.4,
  velocity: 3D $\sim -5/3$, 2D ~ -3

Stratified flow

- We modify the experiment by adding the effect of a stable stratification
- We create an initial density field by combining two constant density fields

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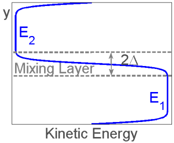

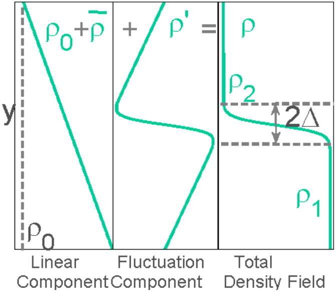

Stratified flow

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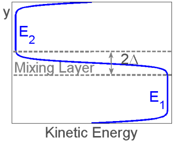
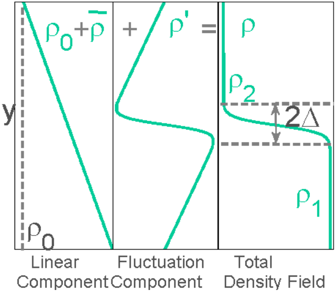
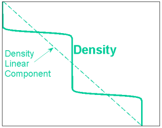




- The fluctuation component has periodic boundary condition $\Rightarrow$ The stability of the stratification is guaranteed



Formulation

Using the Boussinesq approximation the equations that describe the problem are:

$$\nabla \cdot \mathbf{u} = \mathbf{0}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\mathbf{1}}{\rho_0}\nabla \mathbf{p} - \frac{\rho'}{\rho_0}\mathbf{g} + \nu \nabla^2 \mathbf{u}$$

$$\frac{\partial \rho'}{\partial t} + (\mathbf{u} \cdot \nabla)\rho' + \mathbf{v}\frac{\mathbf{d}\rho_{\mathbf{m}}}{\mathbf{dy}} = \mathbf{k}\nabla^2 \mathbf{u}$$

$$\nu = 2.4\ 10^{-10} m^4/s,\ k = 0.3\ 10^{-2},\ Sc* = (\nu/(k * l^2)) = 1.32\ 10^{-4}$$

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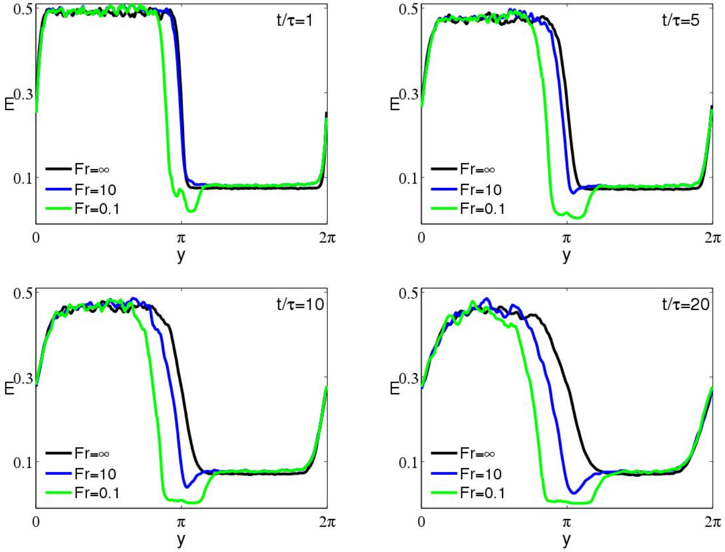
$$\nu = 2.4\ 10^{-10} m^4/s,\ k = 0.3\ 10^{-2},\ Sc* = (\nu/(k * l^2)) = 1.32\ 10^{-4}$$

- The energy ratio is constant, $E_1/E_2 = 6.6$
- The parameter of the experiment is the Froude number

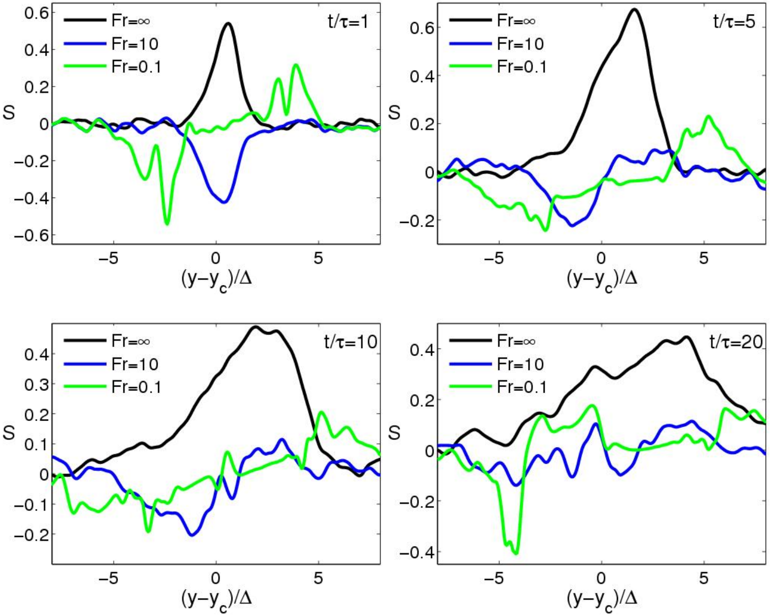
$$Fr = \frac{U}{\sqrt{-\frac{g}{\rho_0}\frac{\partial \rho_m}{\partial y} L}}$$

we considered: $Fr = \infty$ (no stratification), $Fr = 10$ (mild stratification), $Fr = 0.1$ (strong stratification)
movie

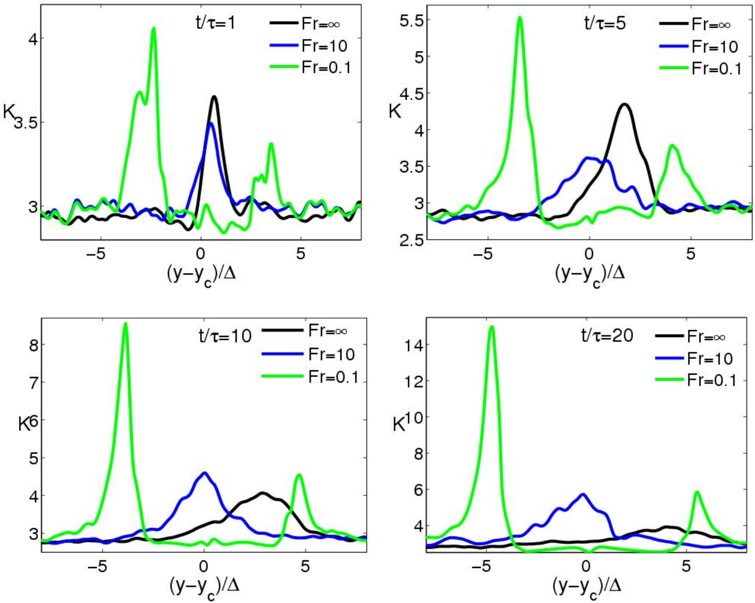
Kinetic Energy



Skewness



Kurtosis


t/τ=1
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=5
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=10
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ
t/τ=20
Fr=∞
Fr=10
Fr=0.1
K
(y−y_c)/Δ

Stratified flow - Main remarks

- For small Froude numbers it is formed a separation layer of zero vorticity
- The energy profile in the mixing region is lower than the minimum value imposed by the initial condition, which shows the effect of the buoyancy force work $\Rightarrow$ Energy hole
- The velocity skewness enlightens the generation of an inverse energy flow and intermittent penetration from the low to the high energy field even in the case of mild stratification