Collective behaviour of linear perturbation wave observed through the energy density spectrum

S. Scarsoglio [1], F. De Santi [2], D. Tordella [2]

1, Department of Water Engineering, Politecnico di Torino, Italy
2, Department of Aeronautics and Space Engineering, Politecnico di Torino, Italy

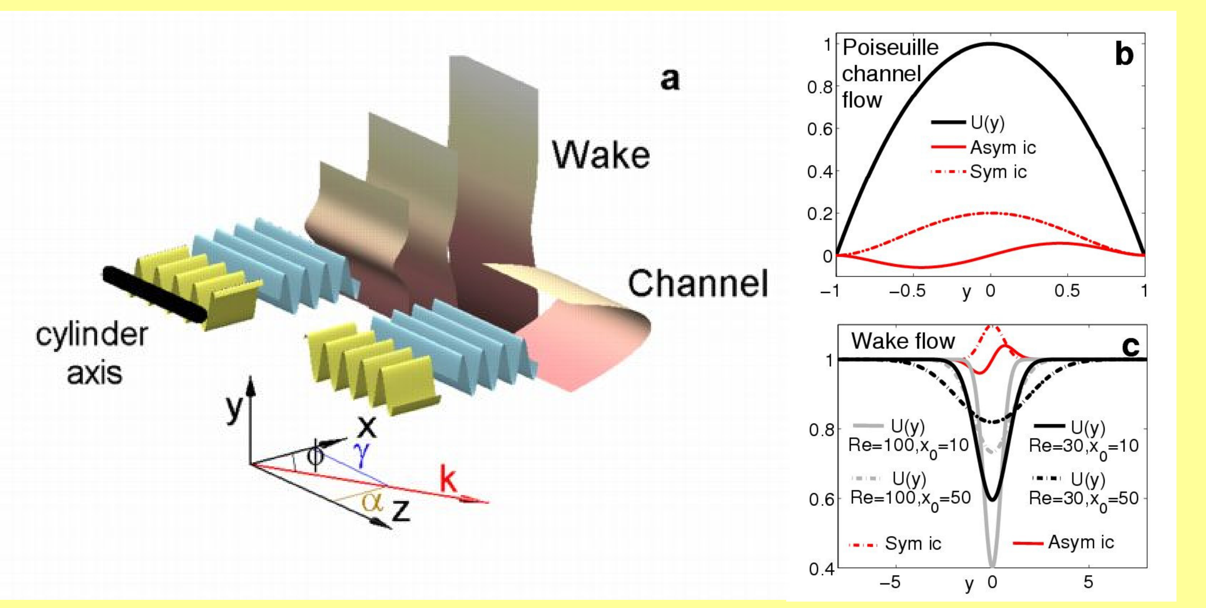
The way in which kinetic energy is distributed over a multiplicity of scales is a fundamental feature of turbulence. According to Kolmogorov's 1941 theory, on the basis of a dimensional analysis, the only possible form of the energy spectrum function is the -5/3 spectrum. Experimental evidence has accumulated to support this law. Until now, this law has been considered a distinctive part of the nonlinear interaction specific to the turbulence dynamics. We show here that this picture is also present in the linear dynamics of three-dimensional stable perturbation waves. Through extensive computation of the transient life of these waves, in typical shear flows, we can observe that the energy they have when they leave the transient phase and enter into the final exponential decay, shows a spectrum that is very close to the -5/3 spectrum. The observation times also show a similar scaling.

Basic flows. Transient computations

We have considered a plane channel flow, as a typical example of wall flow, and plane wake flow, as a typical example of free flow. The base flows for the channel are represented by the Poiseuille solution and for the wake by the first two order terms of the Navier-Stokes asymptotic solution described in [1].

An initial-value problem (IVP) for small arbitrary three-dimensional vorticity perturbations imposed on the basic shear flows is then considered.

The exploration is conducted with respect to physical quantities, such as the polar wavenumber, the angle of obliquity, the symmetry of the perturbation, the flow control parameter, and, for the wake, which is not parallel, the position downstream of the body.



(a) Base flows and perturbation scheme. The flow profiles are qualitatively represented in pink. The perturbation is represented by the blue and the yellow waves which propagate in the x and z directions, respectively. k is the wavenumber, ϕ is its inclination angle with respect to the basic flow. (b)-(c) Initial conditions and velocity profiles. The gray and black lines in the wake panel (c) schematically show the velocity variation with the Reynolds number and the position in the wake.

Collection of transient lives

To measure the growth of perturbations, we define the amplification factor, G, as the kinetic energy density normalized with respect to its initial value,

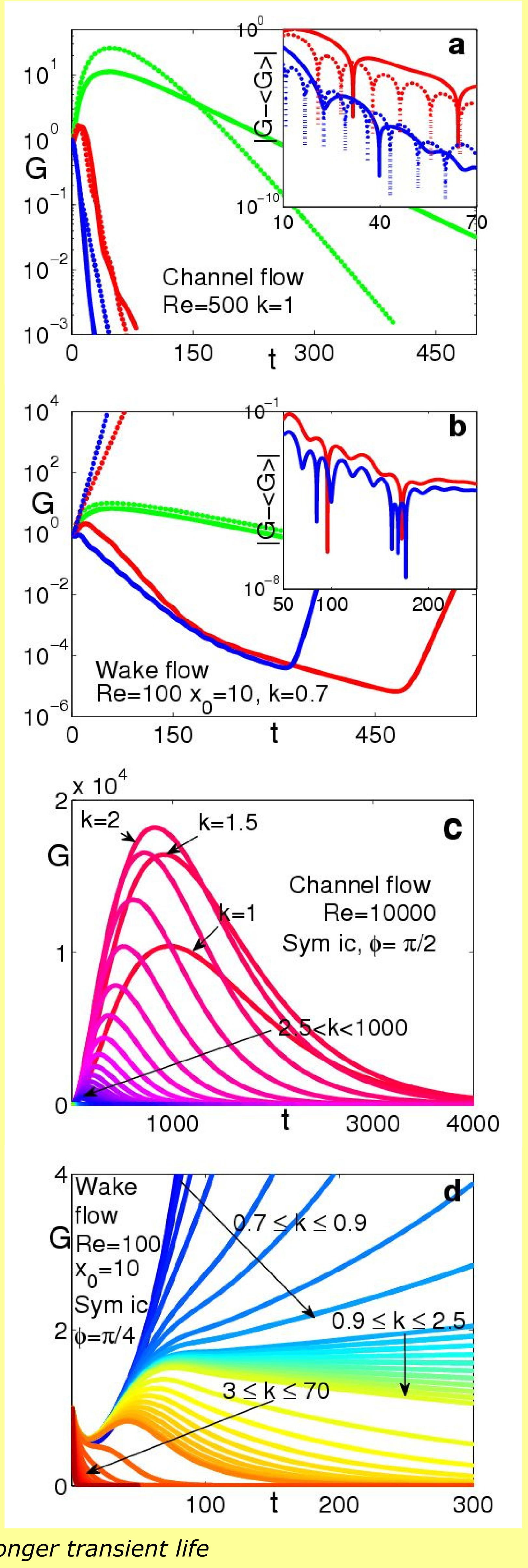
$$G\,(t;\,\alpha\,\gamma)=e\,(t;\,\alpha,\gamma)/e\,(t=0;\,\alpha,\gamma)$$

Where e(t; α, γ) is the kinetic energy density. In terms of amplification factors, the early transient evolution offers very different scenarios for which we present a summary of particular cases.

For example, we have observed two kinds of transients for amplified perturbations, namely a monotone amplification and a growth-decrease-final growth sequence.

In the latter case, if the initial condition is an asymmetric oblique or longitudinal perturbation, the transient clearly shows an initial oscillatory time scale that is associated to a modulation in amplitude of the average value of the pulsation in the early transient (see the insets), and which is different from the asymptotic value of the pulsation [2].

The most important parameters affecting these configurations are the angle of obliquity, the symmetry, and the polar wavenumber. While the symmetry of the disturbance influences the transient behaviour and leaves the asymptotic fate unaltered, a variation in the obliquity.



A collection of transients is presented for both the channel flow and the wake. (a and b): the magnitude of the wavenumber is fixed, while the obliquity and symmetry are allowed to vary. The insets highlight the typical formation of time scales others than the flow external time scale and the wave period. (c and d) shows the transients variation with changes in the wavenumber magnitude. Common trend: long waves can become unstable, but not for any obliquity angles; unstable perturbations with an asymmetric initial condition have a much longer transient life

Spectrum in asymptotic conditions

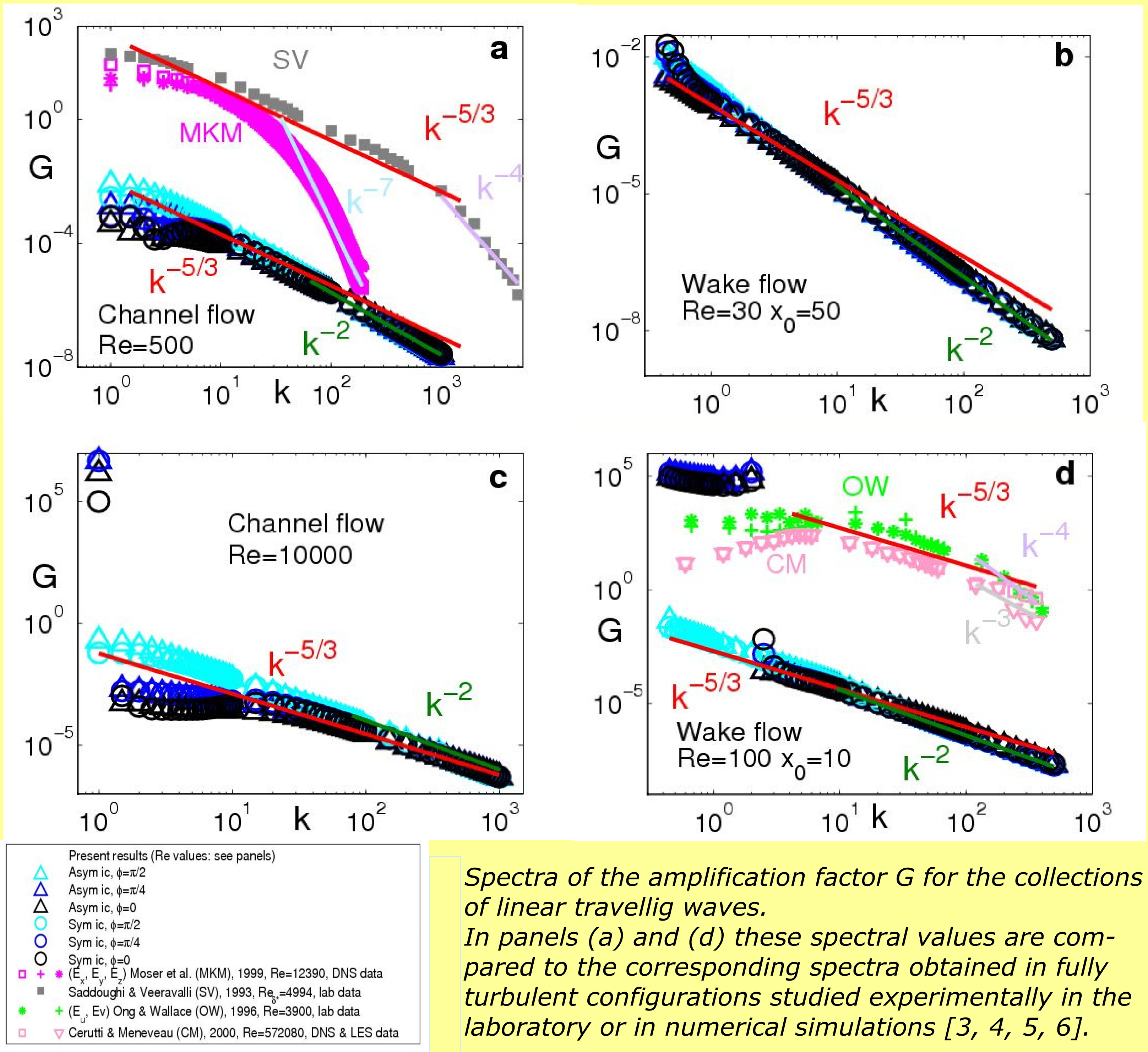
We compare the energy of the waves in correspondence to the transition between the end of the transients and the settlement of the asymptotic condition. Assuming that the temporal asymptotic behaviour of linear perturbations is exponential, the temporal growth rate, *r*, can be defined as

$$r\,(t;\,\alpha,\gamma)=(dG/dt)/G$$

Thus, in an asymptotic condition r is approaching a real constant. Moreover we have selected the instants at which the amplification factor reaches a given rate of variation either in growth or in decay. This situation is represented by the instant, that we call observation time, *Te*, where

$dG/dt < \varepsilon$ or $dG/dt > 1/\varepsilon$, with $\varepsilon = 10^{-n}$.

The present results do not depend on the choice of *n*.

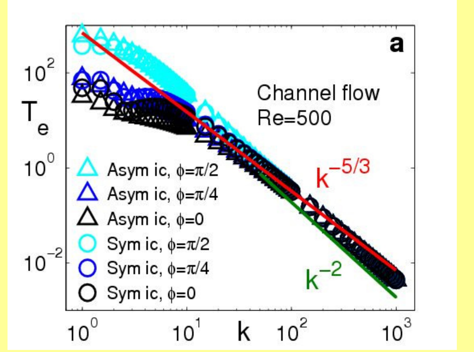


Spectra of the amplification factor G for the collections of linear travellig waves.
In panels (a) and (d) these spectral values are compared to the corresponding spectra obtained in fully turbulent configurations studied experimentally in the laboratory or in numerical simulations [3, 4, 5, 6].

The thus measured spectral values of *G*, show a scaling that is amazingly close to the turbulent canonical value of *-5/3* for the intermediate polar wavenumbers (see figure below).

For shorter wavelengths, characterized by very short transients, the scaling is a little higher in magnitude, approximately equal to *-2*. This result does not appear to be influenced to any great extent by the wave obliquity, the symmetry, or the *Re*.

However, it is possible to observe that purely orthogonal waves show a closer scaling to *-2* than to *-5/3*, even at intermediate wavenumbers.



Spectrum of the observation time

It should be noted that this result appears strengthened: even the observation times, *Te*, present the same scaling. This outcome is not at all trivial. It is sufficient to consider that the observation time includes the transient, and that the different kind of transients we observed are very complex.

Concluding remarks

Spectrum determined by evaluating the energy of the waves when they are exiting their transient state.

Regardless the symmetry and obliquity of perturbations, there exists an intermediate range of wavenumbers in the spectrum where the energy decays with the same exponent observed for fully developed turbulent flows (-5/3), where the nonlinear interaction is considered dominant.

Scale-invariance of G and Te at different (stable and unstable) Reynolds numbers and for different shear flows.

The spectral power-law scaling of inertial waves is a general dynamical property which encompasses the nonlinear interaction.

References

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