



Sudden
transition in
frequency for
travelling waves

S. Scarsoglio,
F. De Santi,
D. Tordella

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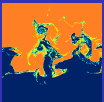
Travelling perturbations in sheared flows: sudden transition in frequency and phase speed asymptotics

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Francesca De Santi Daniela Tordella

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General aspects

- Temporal evolution of the wave frequency in two archetypical shear flows, the plane channel flow and the bluff-body wake;

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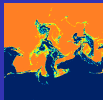
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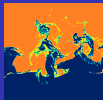
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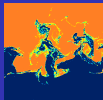
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⇒ Attention mainly devoted to the frequency of vortex shedding for the most unstable spatial scales (Williamson, 1989; Strykowski & Sreenivasan, 1990);





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 - High maxima of energy followed by asymptotic damping;



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- ⇒ **How does the frequency of these travelling waves behave?**



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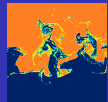
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 - High maxima of energy followed by asymptotic damping;
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- ⇒ **How does the frequency of these travelling waves behave?**
- Nonlinear terms limit the amplitude of the wave packet leaving unaffected its frequency (Delbende & Chomaz, 1998).



Perturbation scheme

- Steady base flow excited with small 3D perturbations;

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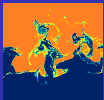
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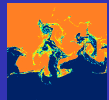
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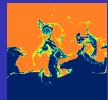
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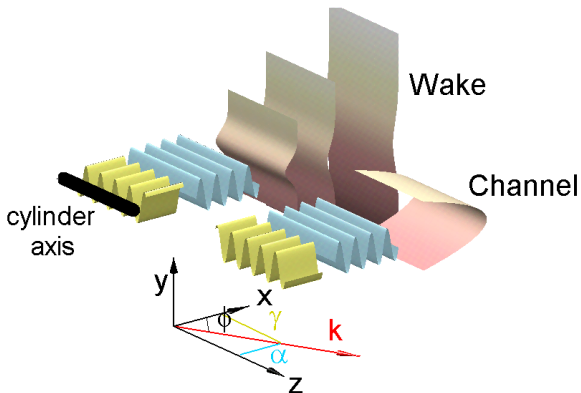
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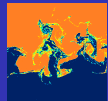
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- Kinetic energy density e :

$$e(t; \alpha, \gamma) = \frac{1}{2} \int_{-y_d}^{+y_d} (|\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2) dy$$



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$$G(t; \alpha, \gamma) = \frac{e(t; \alpha, \gamma)}{e(t=0; \alpha, \gamma)}$$



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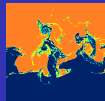
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- Amplification factor G :

$$G(t; \alpha, \gamma) = \frac{e(t; \alpha, \gamma)}{e(t=0; \alpha, \gamma)}$$

- Temporal growth rate r :

$$r(t; \alpha, \gamma) = \frac{\log[e(t; \alpha, \gamma)]}{2t}$$





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- Phase θ_w :

$$\theta_w(y, t; \alpha, \gamma) = \arg(\hat{v}(y, t; \alpha, \gamma))$$



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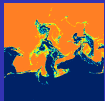
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- Phase θ_w :

$$\theta_w(y, t; \alpha, \gamma) = \arg(\hat{v}(y, t; \alpha, \gamma))$$

- Angular frequency ω :

$$\omega(t; y_0, \alpha, \gamma) = \frac{|d\theta(t; y_0, \alpha, \gamma)|}{dt}$$





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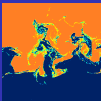
- Angular frequency ω :

$$\omega(t; y_0, \alpha, \gamma) = \frac{|d\theta(t; y_0, \alpha, \gamma)|}{dt}$$

- Phase velocity \mathbf{C} :

$$\mathbf{C} = (\omega/k)\hat{\mathbf{k}}$$

$\hat{\mathbf{k}} = (\cos(\phi), \sin(\phi))$ is the unitary vector in the k direction.





Channel flow ($Re = 10000, k = 15, y_0 = 0.5$)

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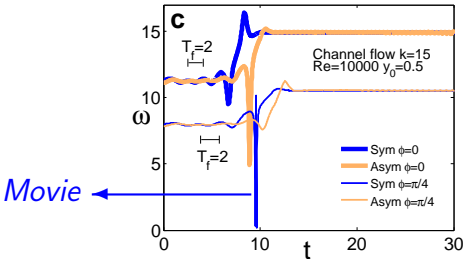
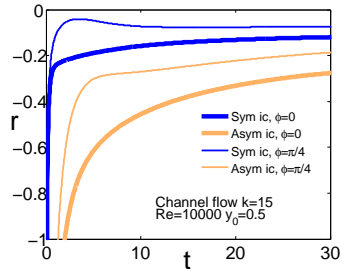
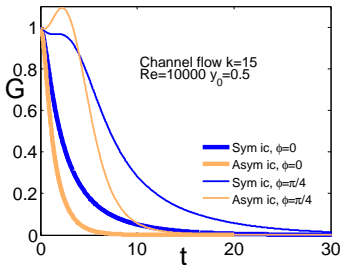
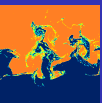
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Wake flow ($Re = 100, k = 0.7, x_0 = 50, y_0 = 1$)

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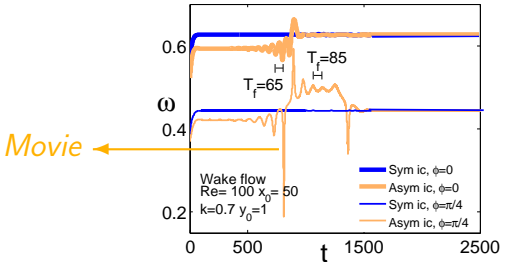
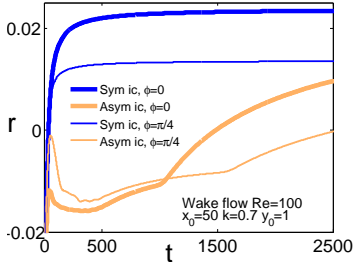
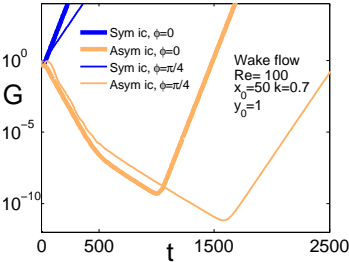
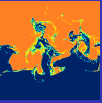
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Movie ←



Typical transient time scales

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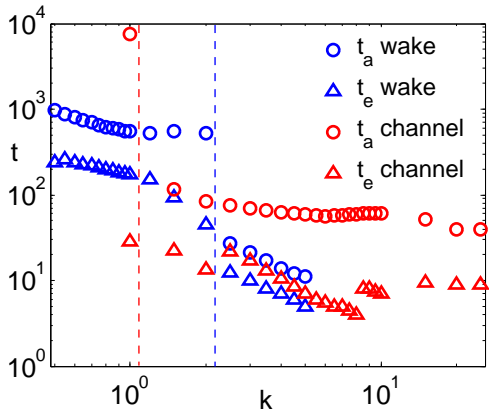
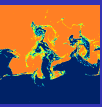
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t_a (\circ): the asymptotic limit is reached. t_e (\triangle): the early transient ends and the frequency discontinuities occur. **Blue symbols:** wake flow, $Re = 100$, antisymmetric input, $\phi = \pi/4$, $x_0 = 10$, $y_0 = 1$. **Red symbols:** channel flow $Re = 10000$, symmetric input, $\phi = \pi/4$, $y_0 = 0.5$.



Spectra of the frequency and phase velocity

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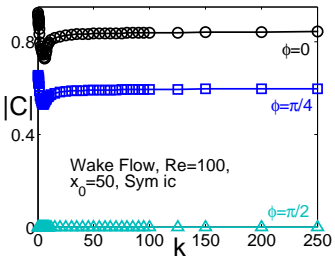
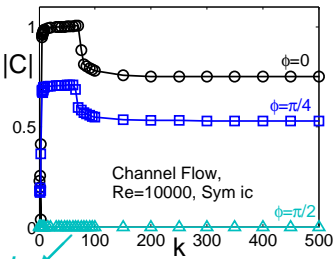
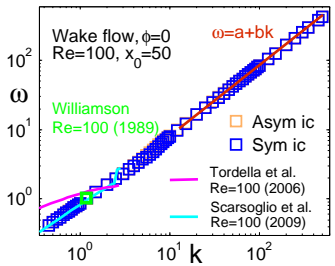
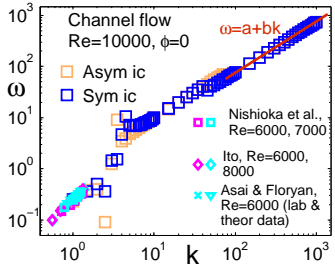
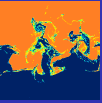
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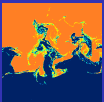
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- Discontinuities in frequency **never observed before** (loss of information with the spectral analysis of signals);





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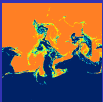
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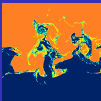
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- **Intermediate transient**;
- Orthogonal standing waves \Rightarrow **How does a spot form?**



Coming next...Spot formation

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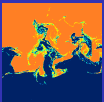
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- Single wave: $k = 0.6$;





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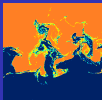
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- Different wavelengths:
 $k \sim 10^{-1} - 10^0$;





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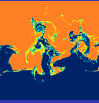
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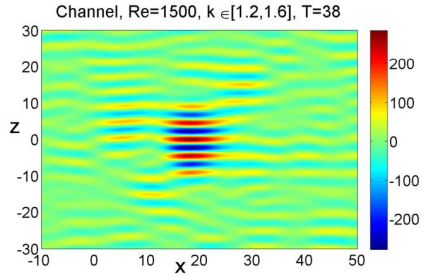
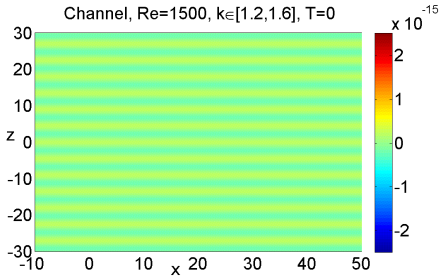
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- Single wave: $k = 0.6$;
- Different wavelengths: $k \sim 10^{-1} - 10^0$;
- Wave packet: $k \sim 1.4$.





Influence of the Reynolds number

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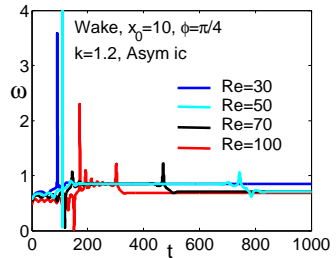
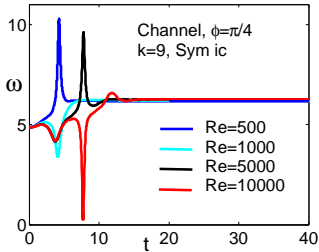
Transient dynamics

Jumps in frequency
Intermediate term

Asymptotic behaviour

Dispersion relation

Conclusions





Different transversal observation points y_0

Sudden transition in frequency for travelling waves

S. Scarsoglio, F. De Santi, D. Tordella

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