

Small Scale Anisotropy and other results in Turbulent Shear-less Mixing

**Daniela Tordella
Politecnico di Torino**

**The Nature of Turbulence Program
Kavli Insitute, University of California Santa
Barbara, March 24, 2011.**

Step onset from an initial uniform distribution of turbulent energy

12th European Turbulence Conference, Marburg, September 2009

Daniela Tordella, Michele Iovieno

*Dipartimento di Ingegneria Aeronautica e Spaziale
Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129 Torino, Italy*



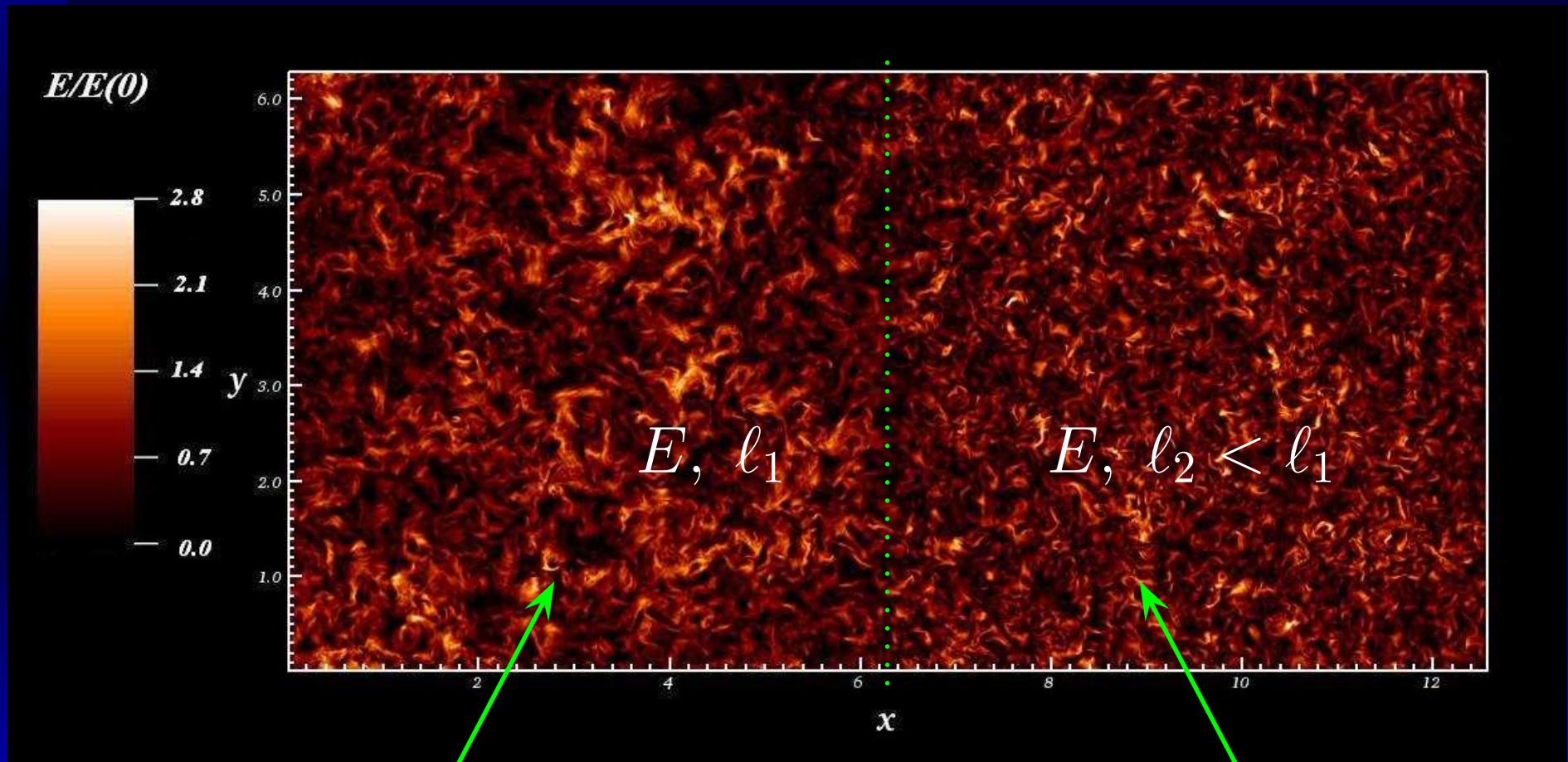
An integral scale gradient introduced
in a uniform kinetic energy
distribution can generate:

- an energy gradient
- a highly intermittent layer



Flow Configuration

Initially uniform turbulent kinetic energy:

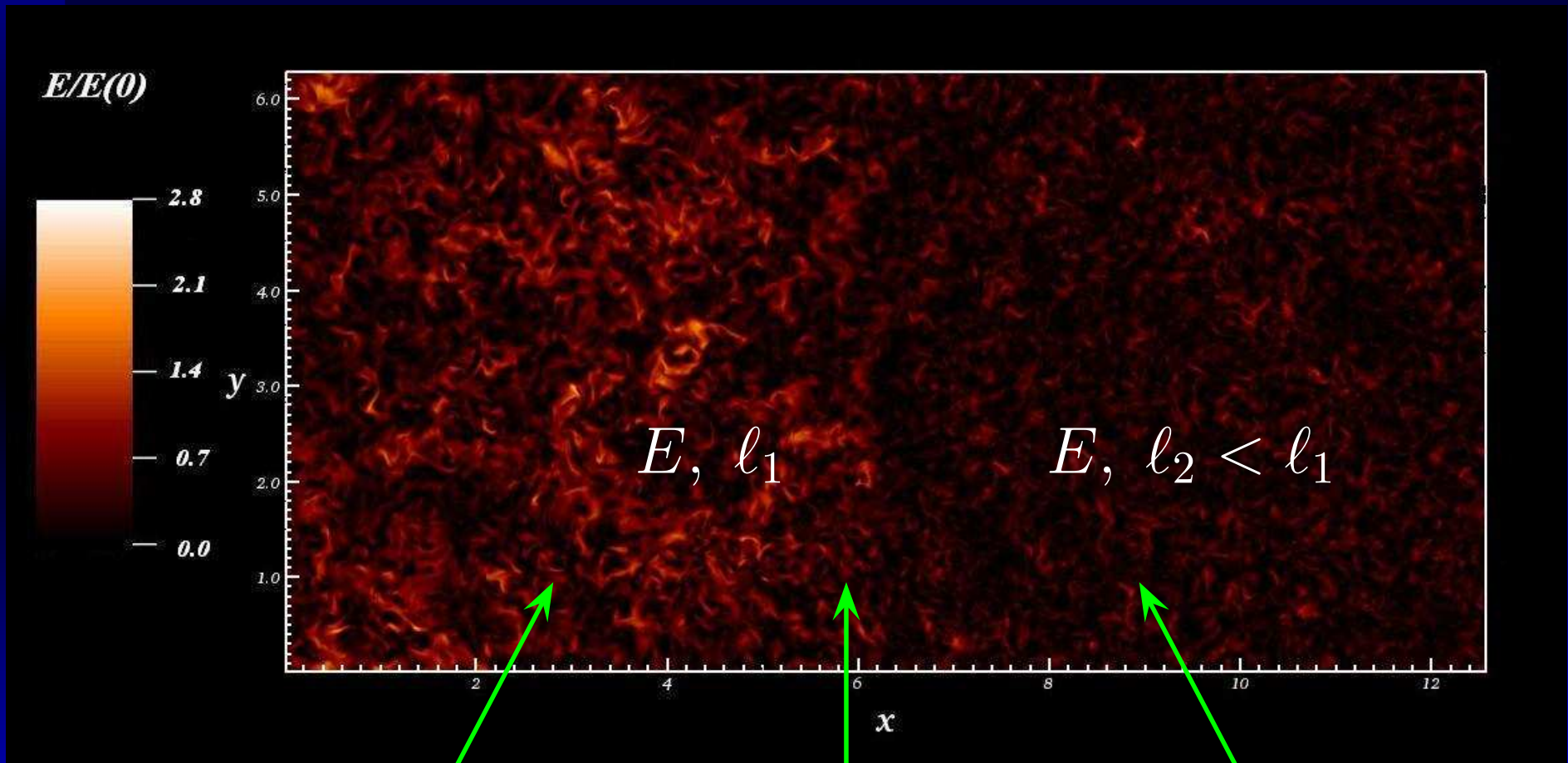


1-Larger scale turbulence 2-Smaller scale turbulence



Flow Configuration

Initially uniform turbulent kinetic energy:

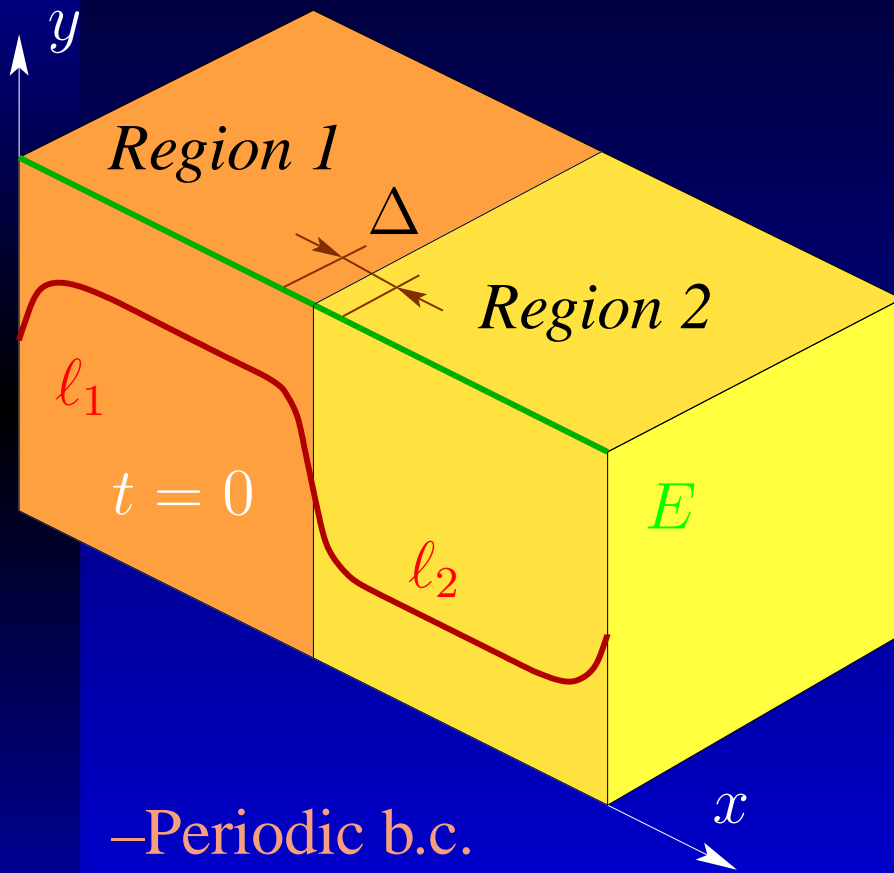


1-Larger scale turbulence 2-Smaller scale turbulence

Shearless mixing layer



Method



–Periodic b.c.

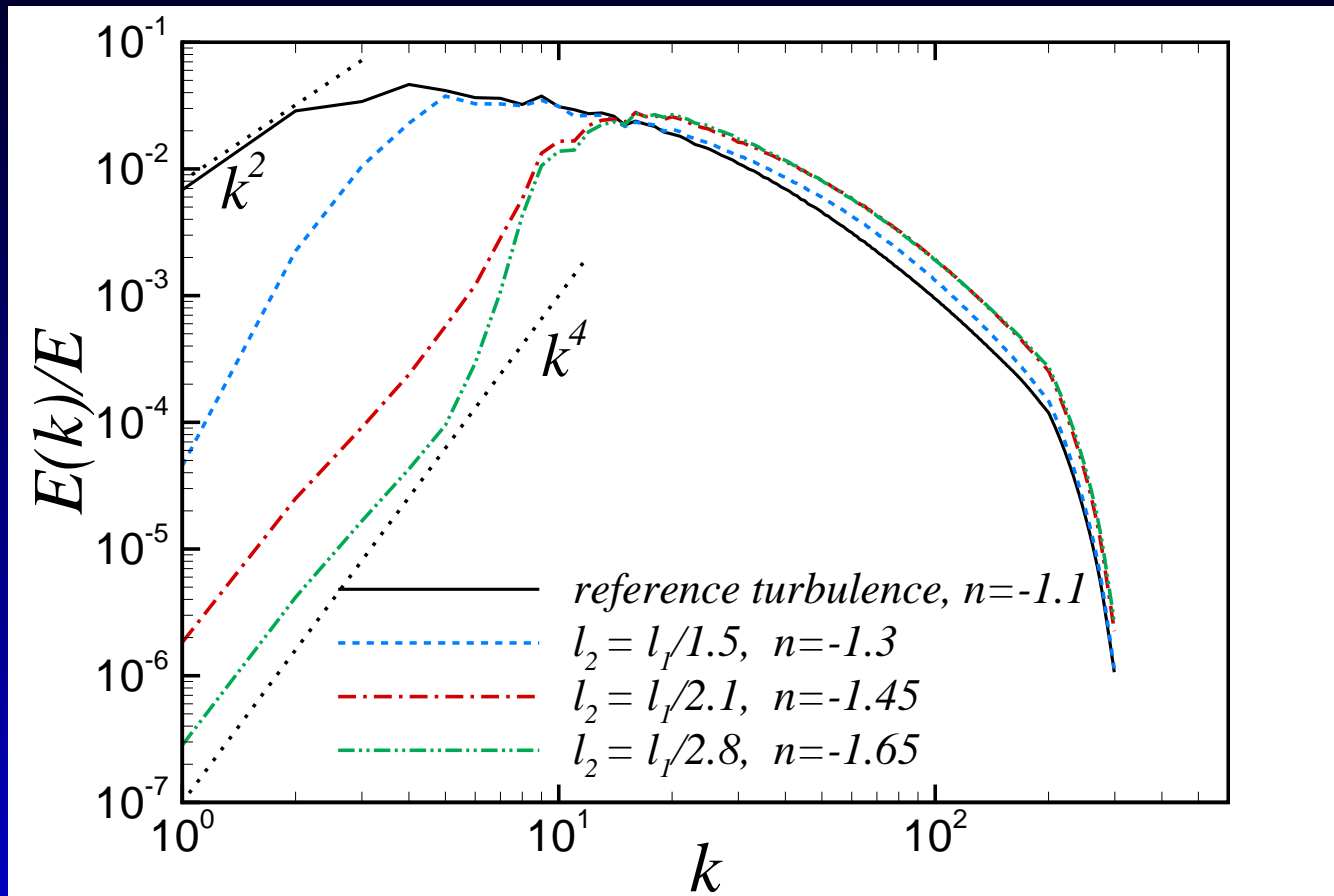
–Temporal decay

- **DNS:**

- ▶ $Re_\lambda = 150$
- ▶ parallelepiped domain,
 $2\pi \times 2\pi \times 4\pi$
- ▶ $600^2 \times 1200$ grid points
- ▶ Fourier-Galerkin pseudospectral space discretization
- ▶ explicit RK-4 time integration



Initial energy spectra

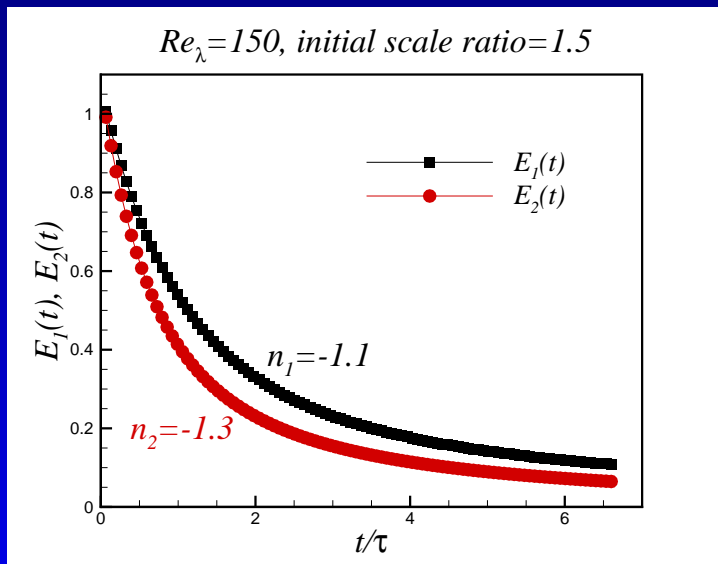
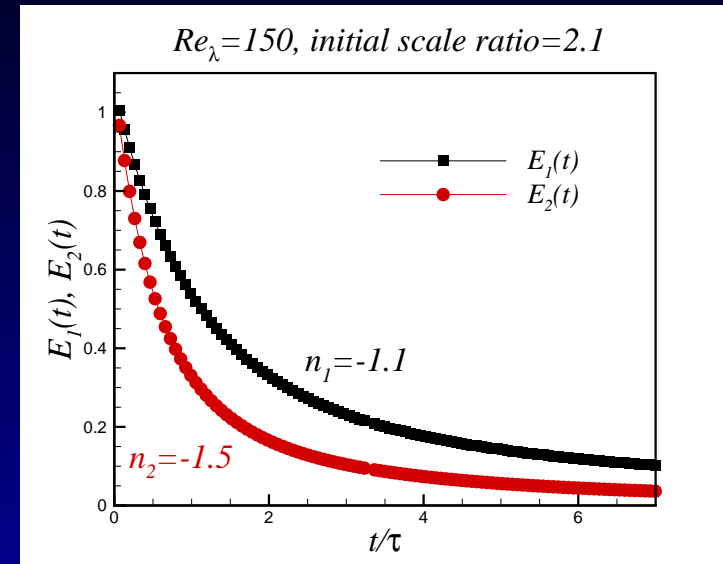
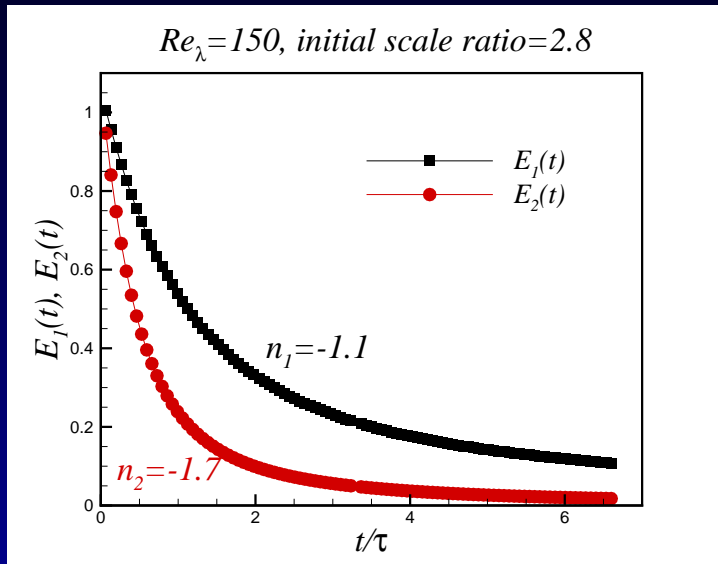


Field 1 \rightarrow larger integral scale

Field 2 \rightarrow smaller integral scale



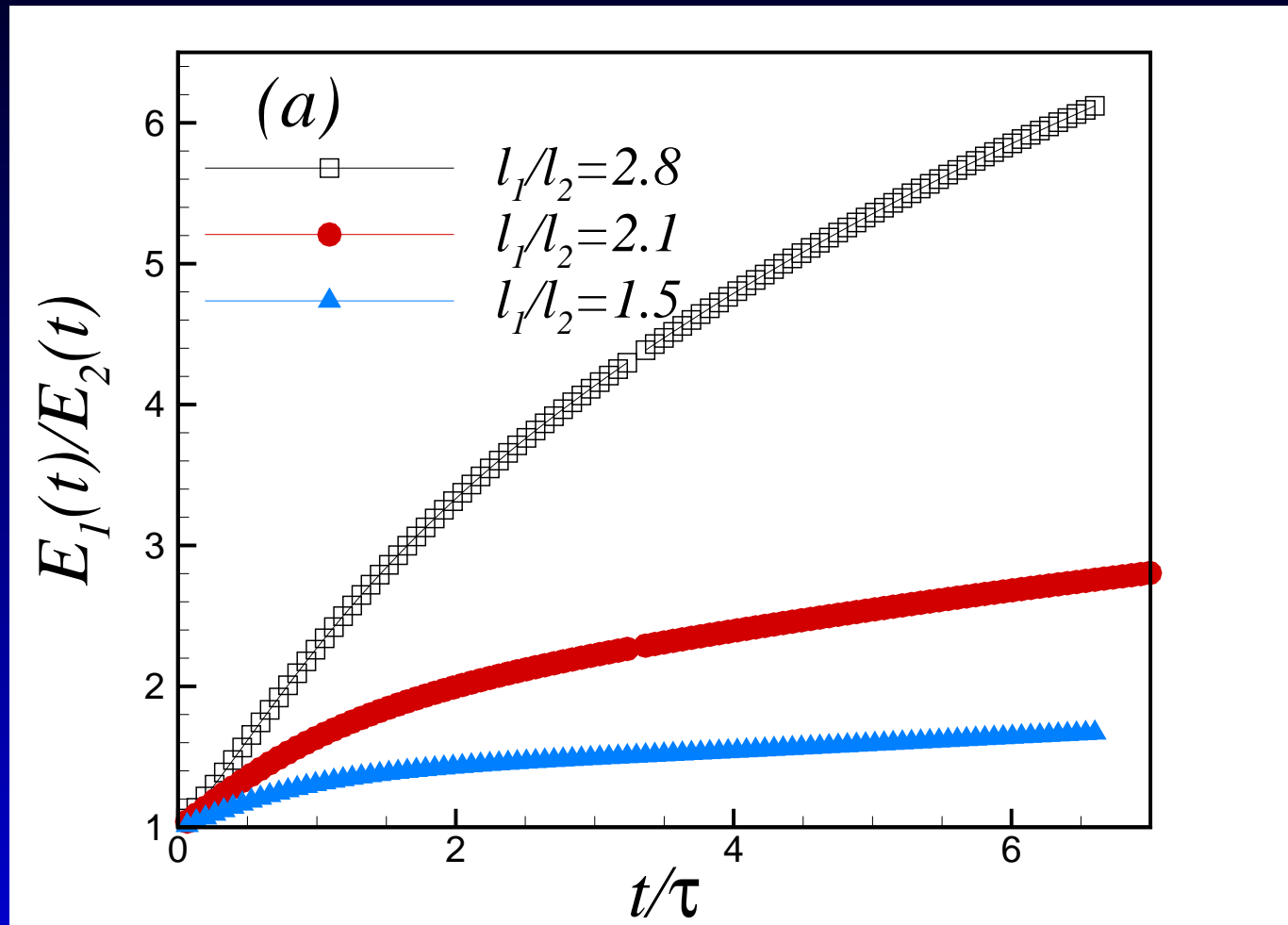
Turbulent kinetic energy decay



Homogenous turbulence
with smaller scale de-
cays faster
 \Rightarrow a kinetic energy gra-
dient is generated



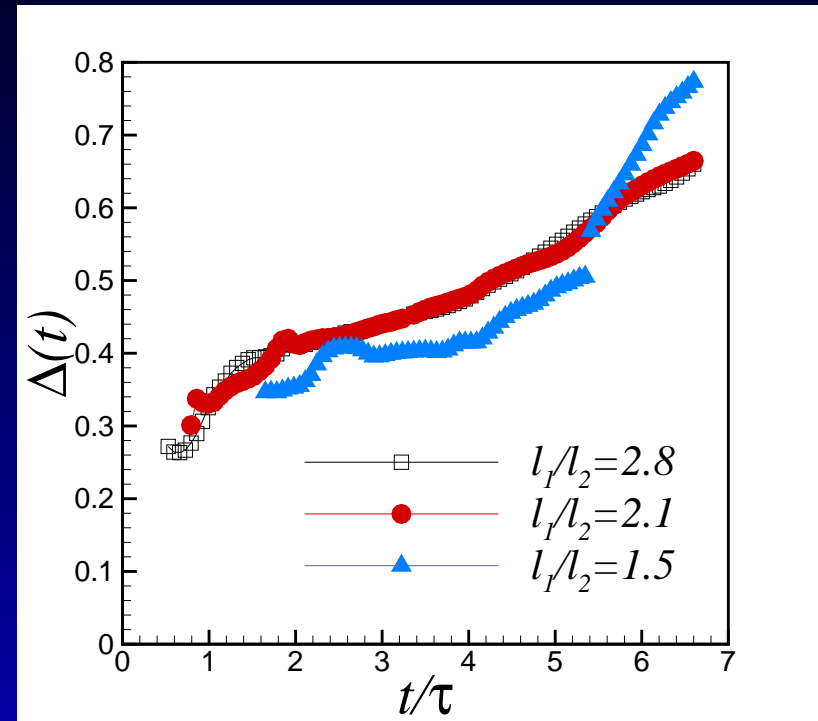
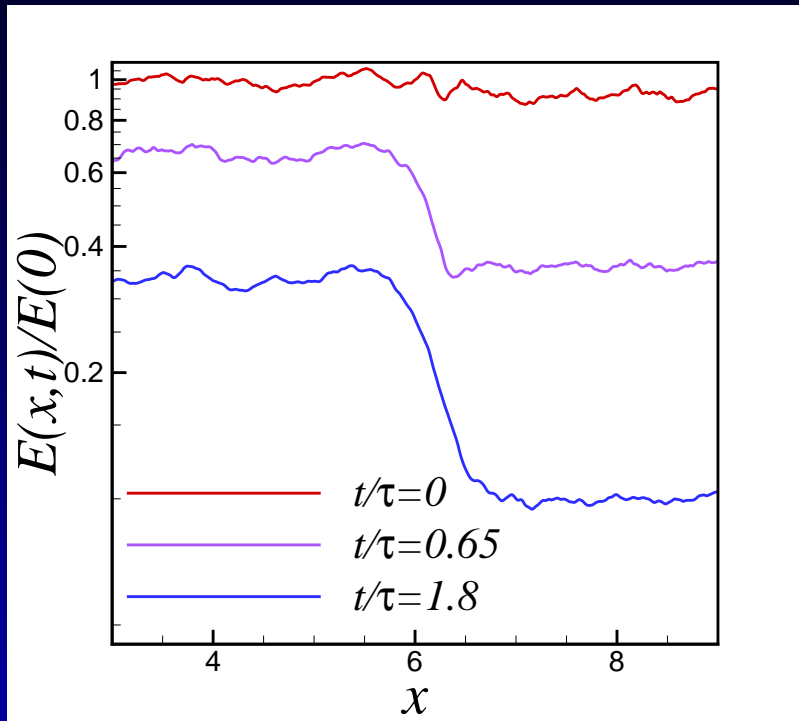
Energy Ratio



Time evolution of the energy ratio E_1/E_2 .



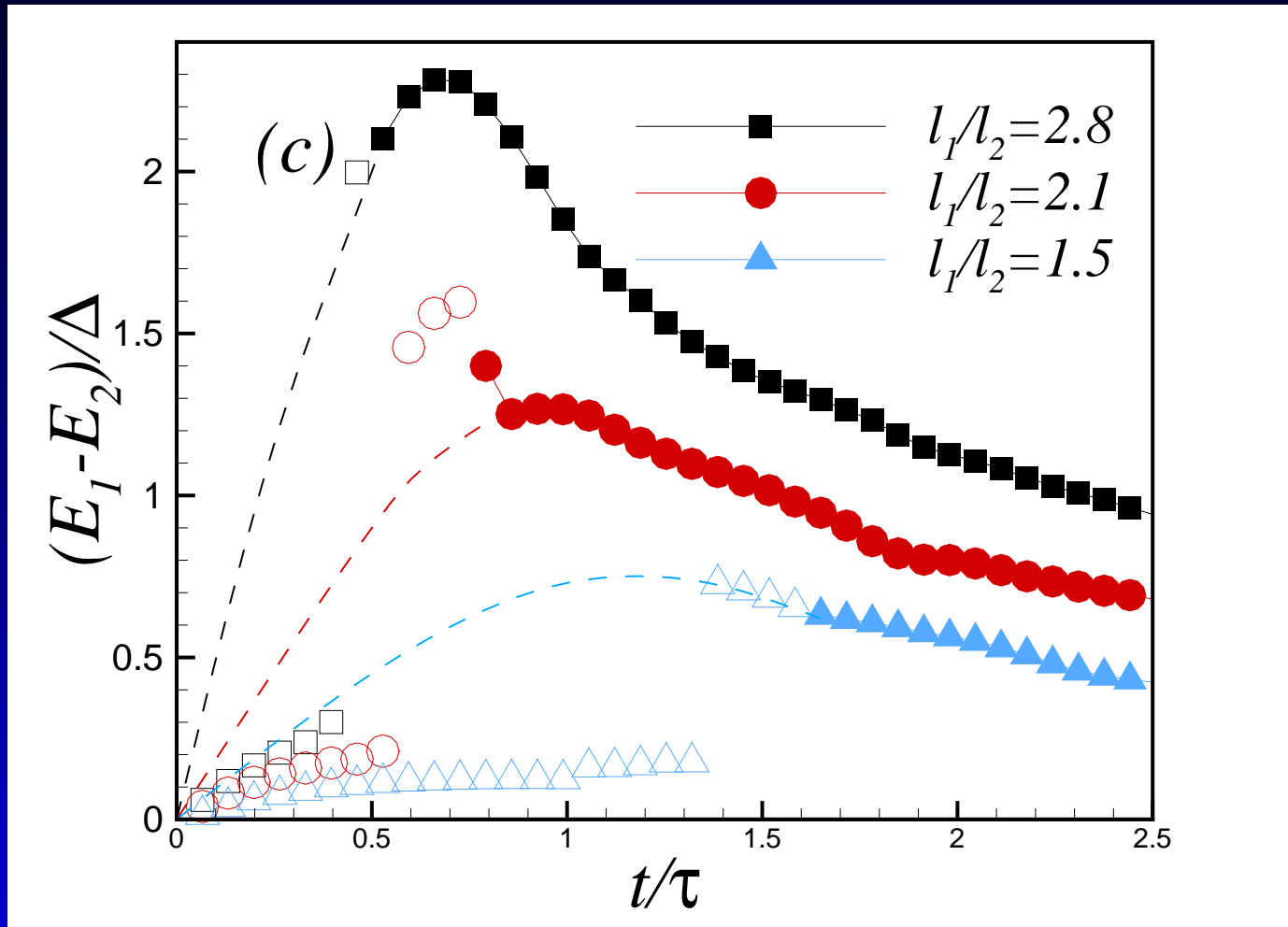
Mixing layer thickness $\Delta(t)$



$\tau =$ initial eddy turnover time

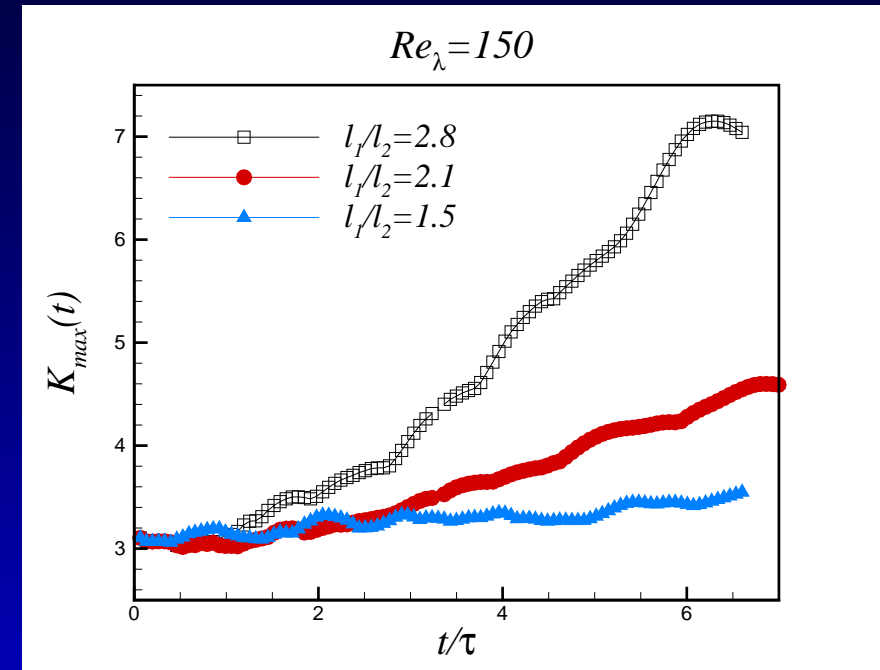
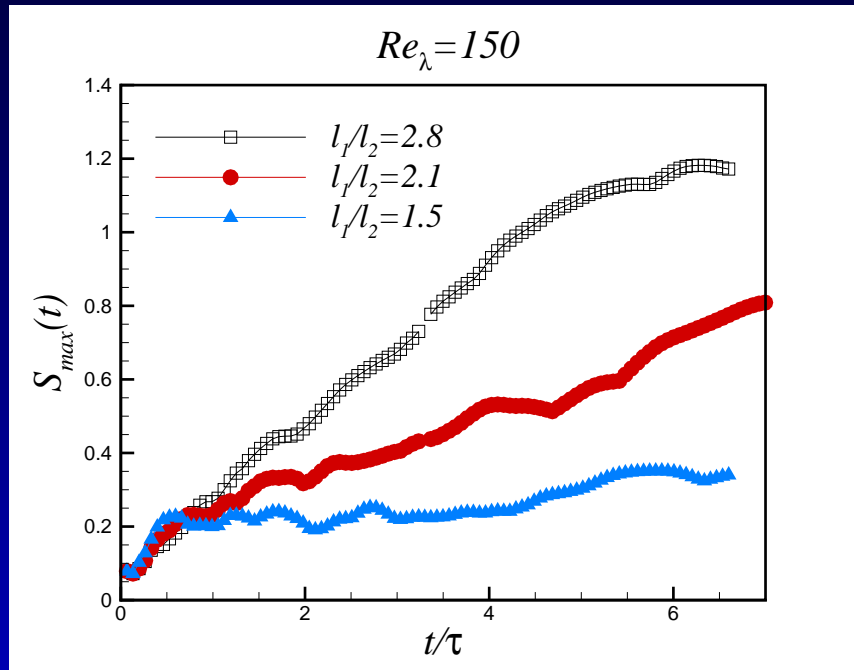


Kinetic energy gradient

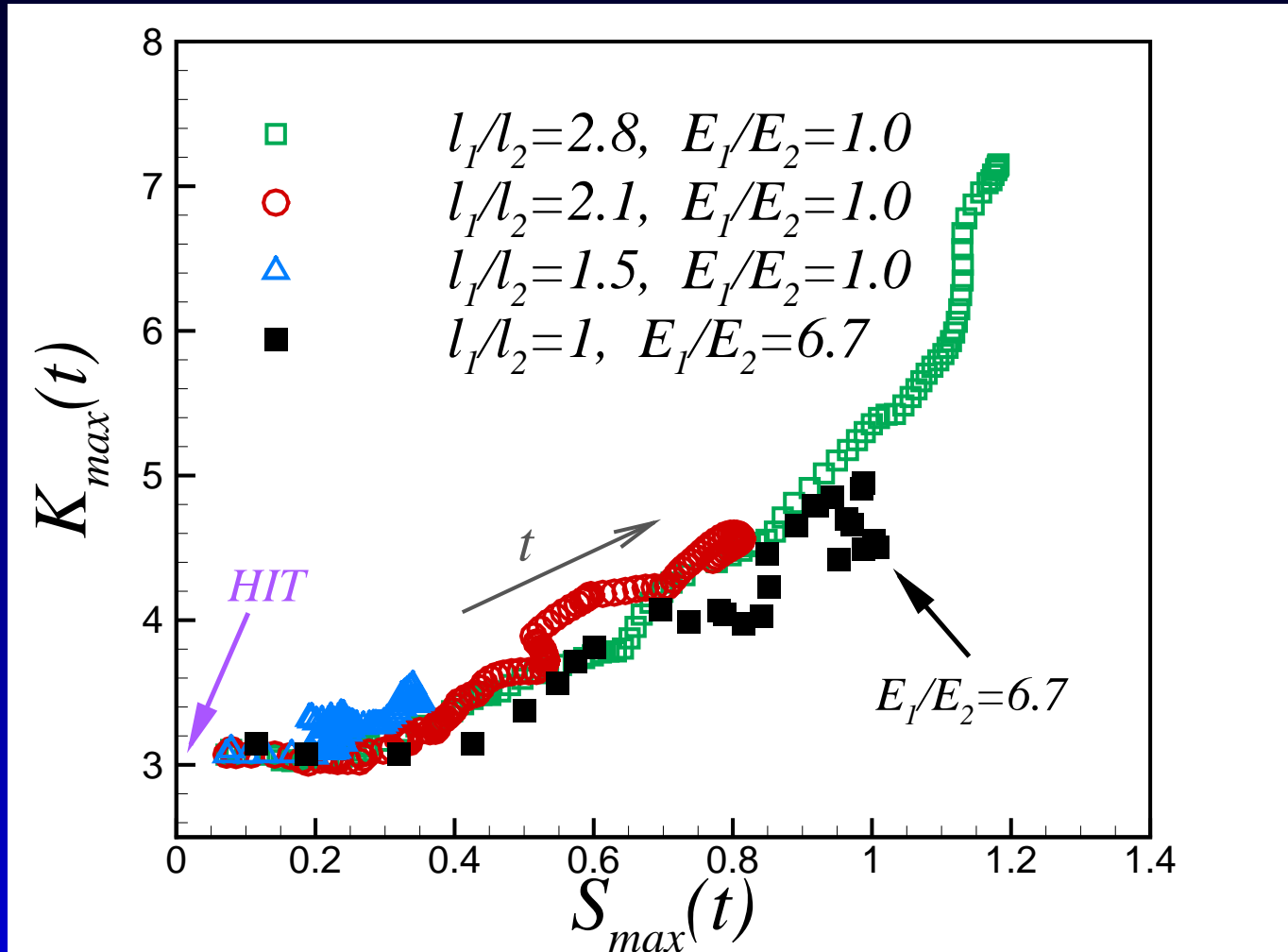


Mixing layer intermittency

Velocity skewness and kurtosis, component in the inhomogeneous direction: maximum in the mixing layer



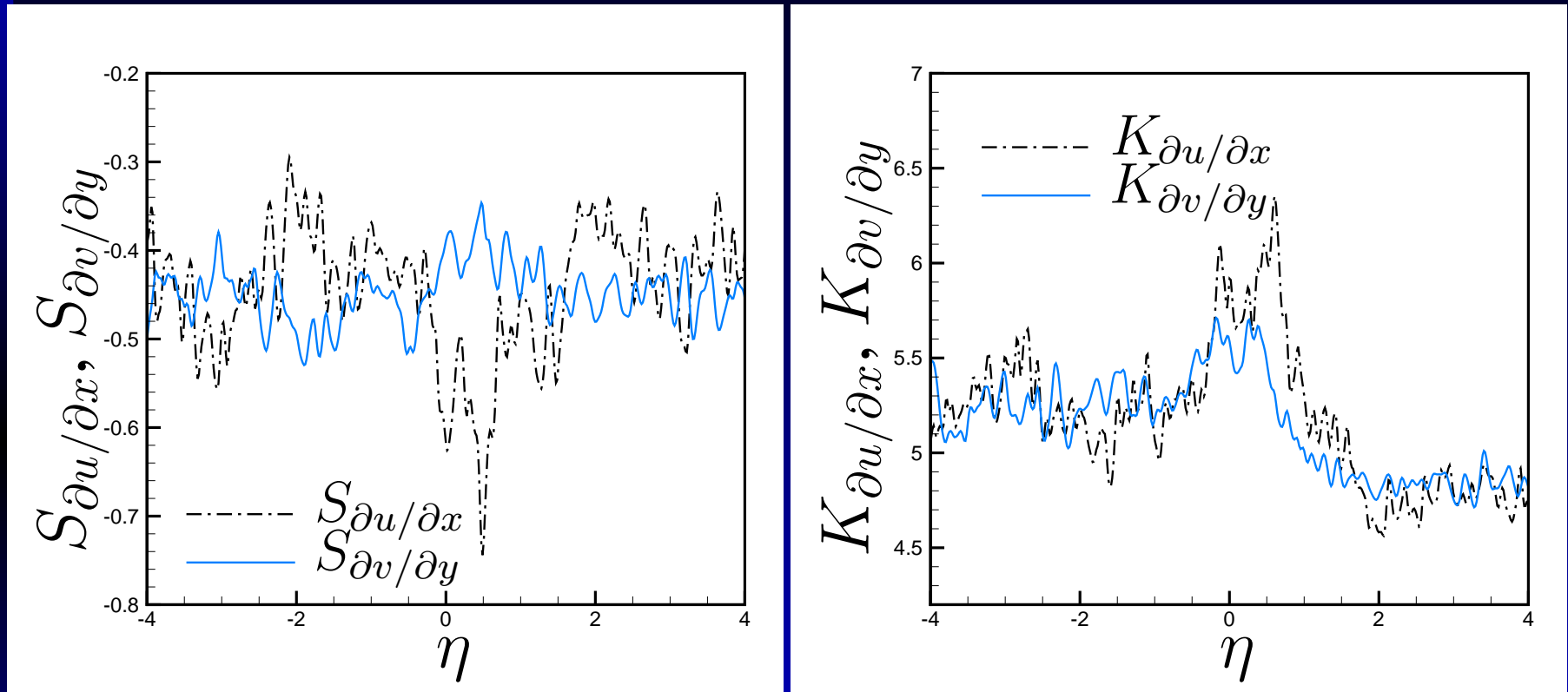
Intermittency



A scale gradient can generate more intermittency than an energy gradient in presence of a uniform scale



Longitudinal derivatives



Spatial distribution of longitudinal moments,

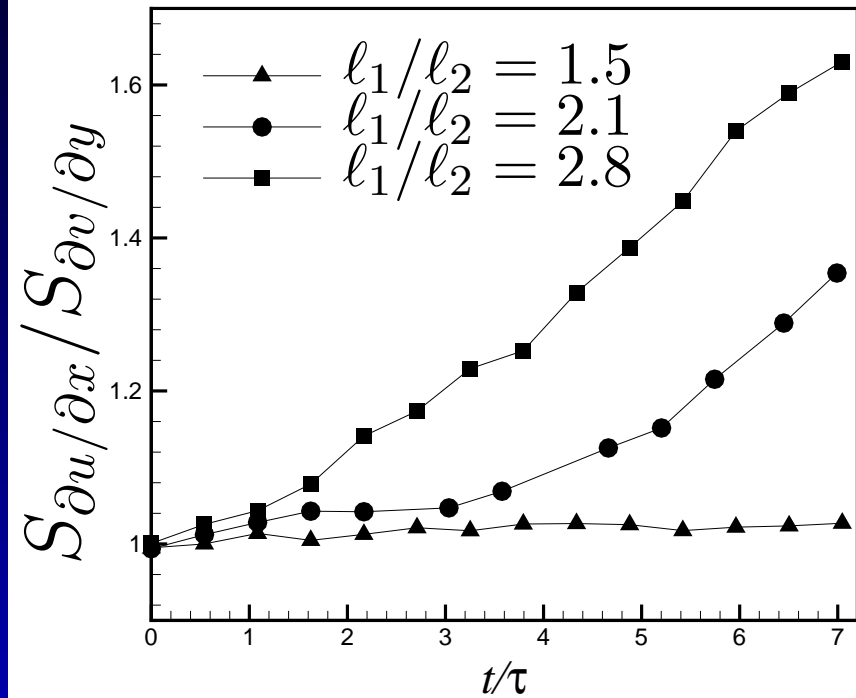
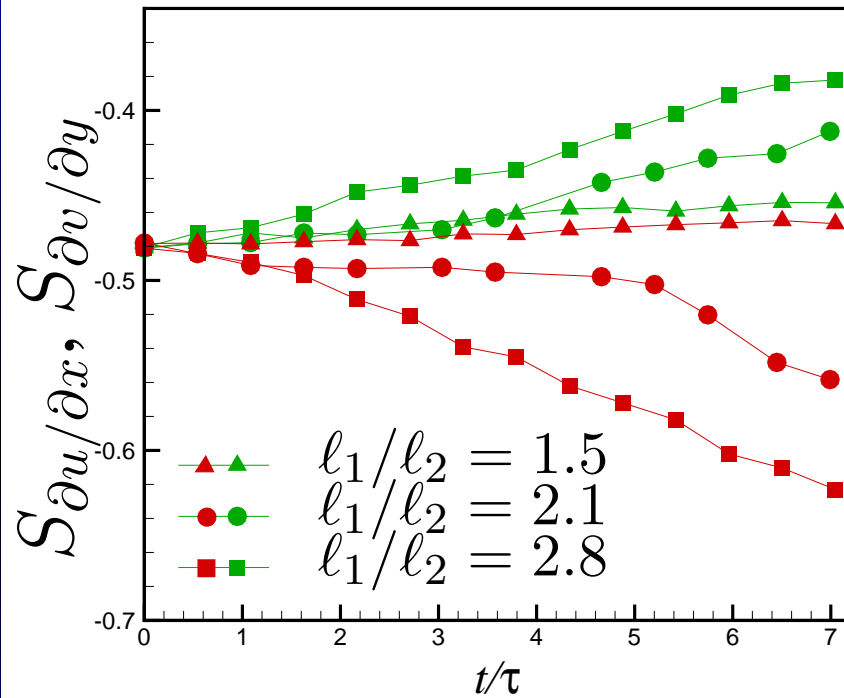
$$\eta = x/\Delta,$$

x, u in the inhomogenous direction,

y, v in homogenous directions.



Longitudinal derivatives



Anisotropy is propagated to small scales.



Conclusions

Simulations of a flow with an homogenous energy and an integral scale gradient show:

- an integral scale inhomogeneity generates an energy gradient
- the decay exponent of turbulent flow with the same initial energy depends on their integral scale \Rightarrow the smaller the scale, the faster the decay.
- intermittency can be higher than that generated by an energy gradient and a uniform scale
- anisotropy and intermittency quickly spread to small scales.



Small scale anisotropy induced by a spatial variation of the integral scale

Euromech Colloquium 512, Torino, October 2009

Daniela Tordella, Michele Iovieno

Dipartimento di Ingegneria Aeronautica e Spaziale

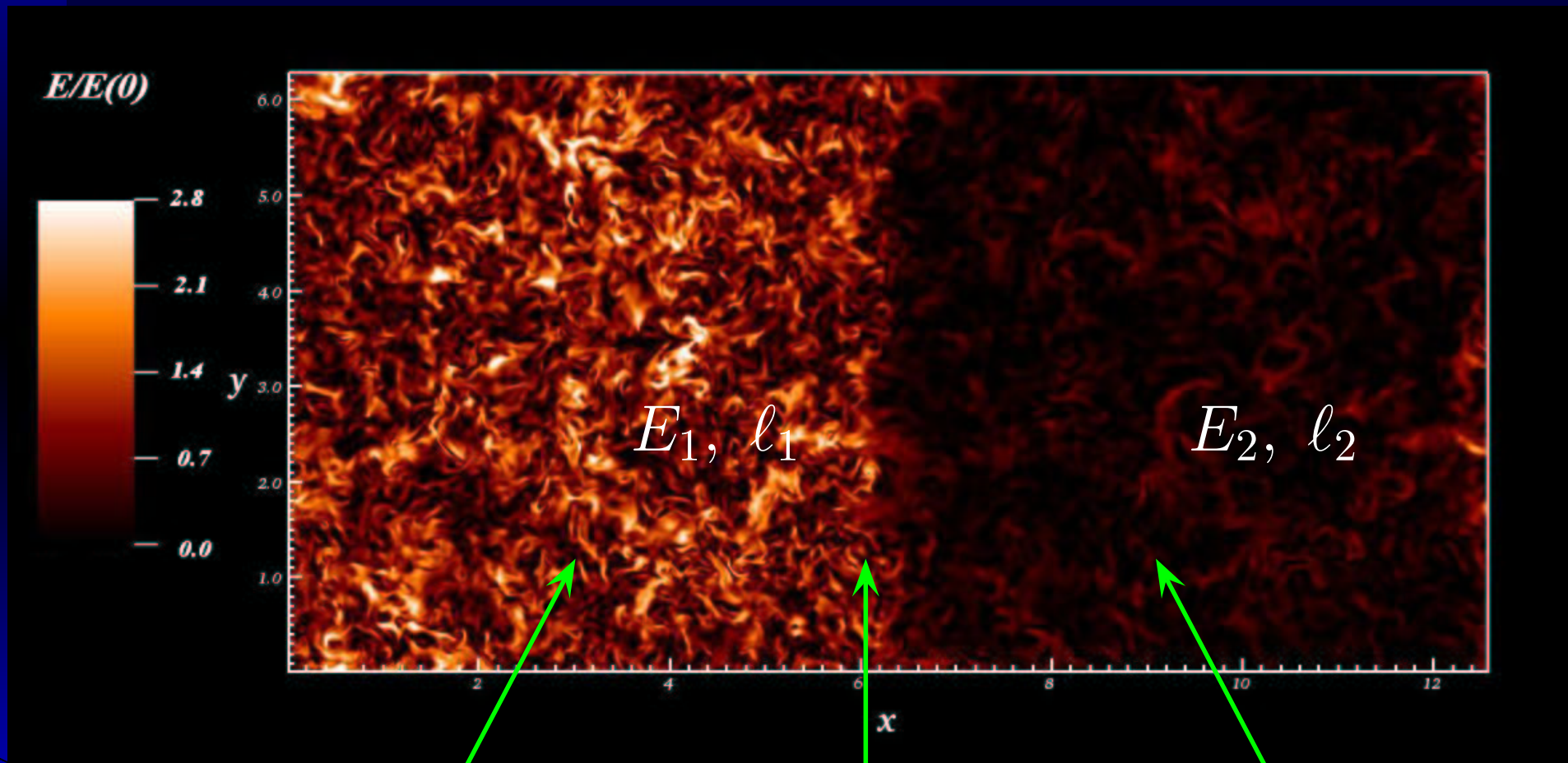
Politecnico di Torino,

Corso Duca degli Abruzzi 24, 10129 Torino, Italy



Turbulent shearless mixing

$$Re_\lambda = 150, E_1/E_2 = 6.6, t/\tau = 0.92$$



Homogeneous region 1

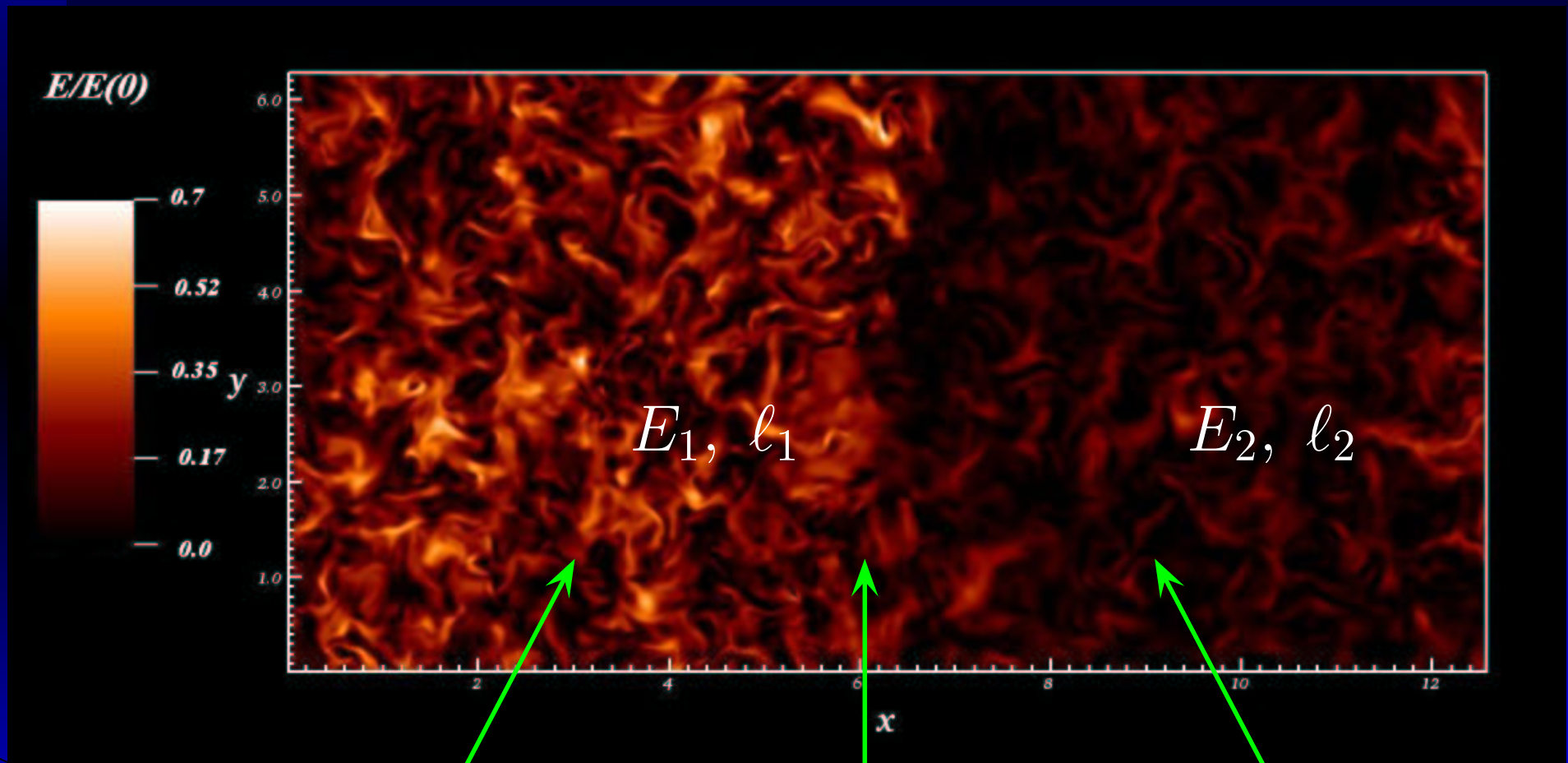
Homogeneous region 2

Mixing layer



Turbulent shearless mixing

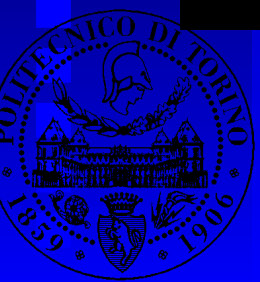
$$Re_\lambda = 150, E_1/E_2 = 6.6, t/\tau = 6.7$$



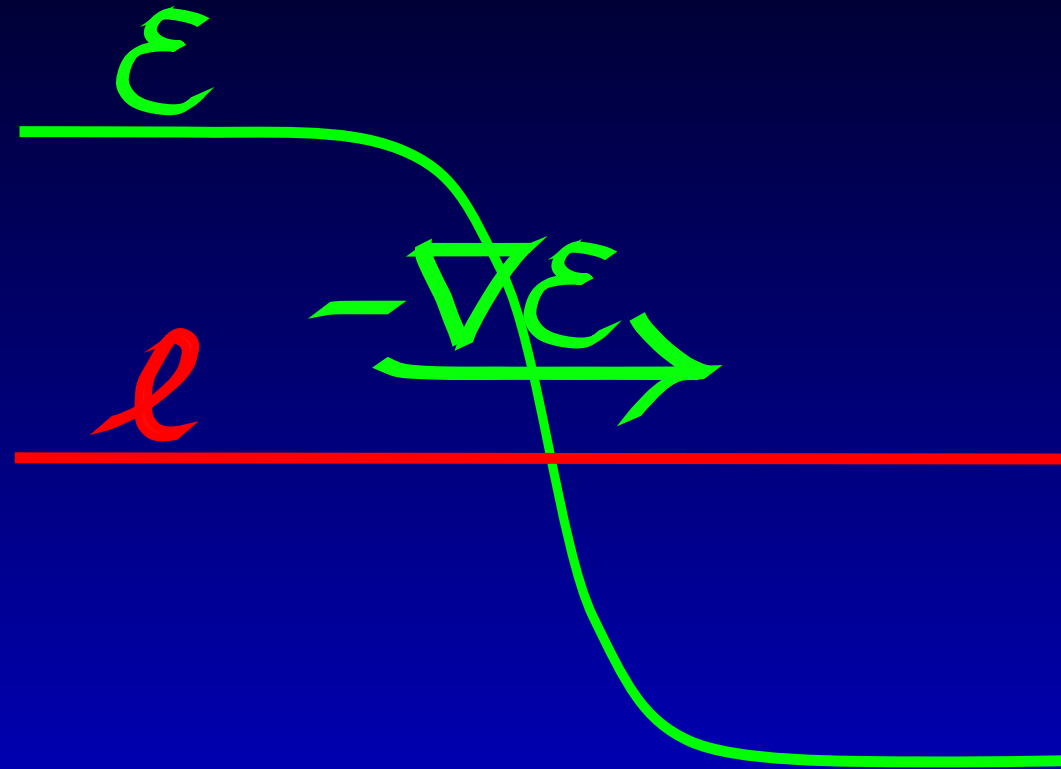
Homogeneous region 1

Homogeneous region 2

Mixing layer



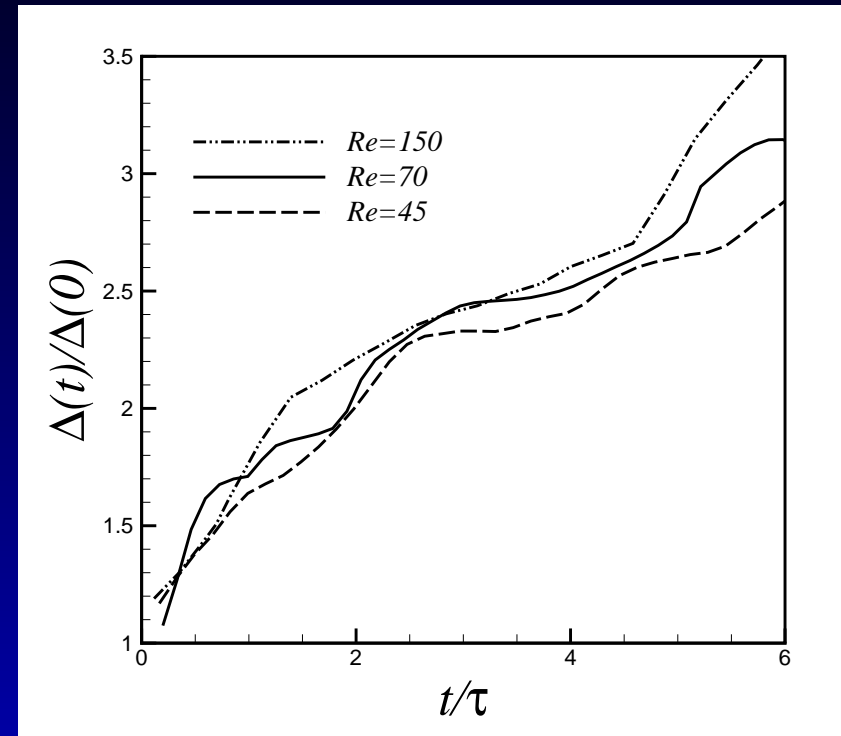
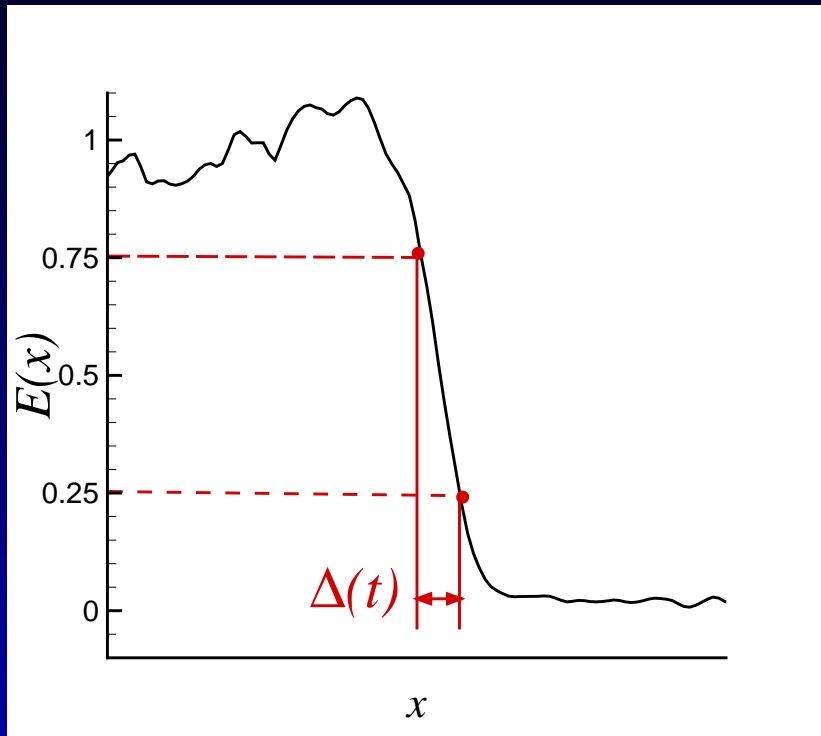
1-Gradient of energy



Gradient of energy, uniform integral scale



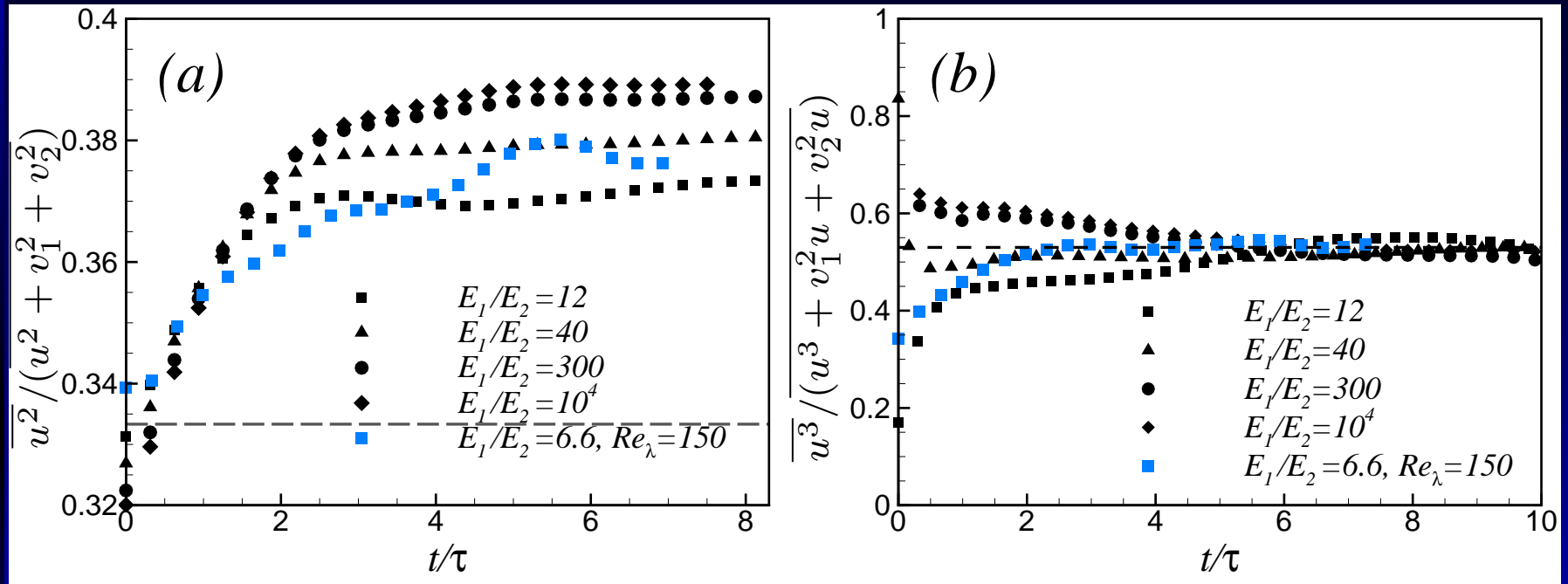
Mixing layer thickness, $E_1/E_2 = 6.7$



$\Delta(t)$ is the mixing layer thickness, defined from the kinetic energy distribution, see *JFM* 2006, $\Delta \sim t^{0.45}$.



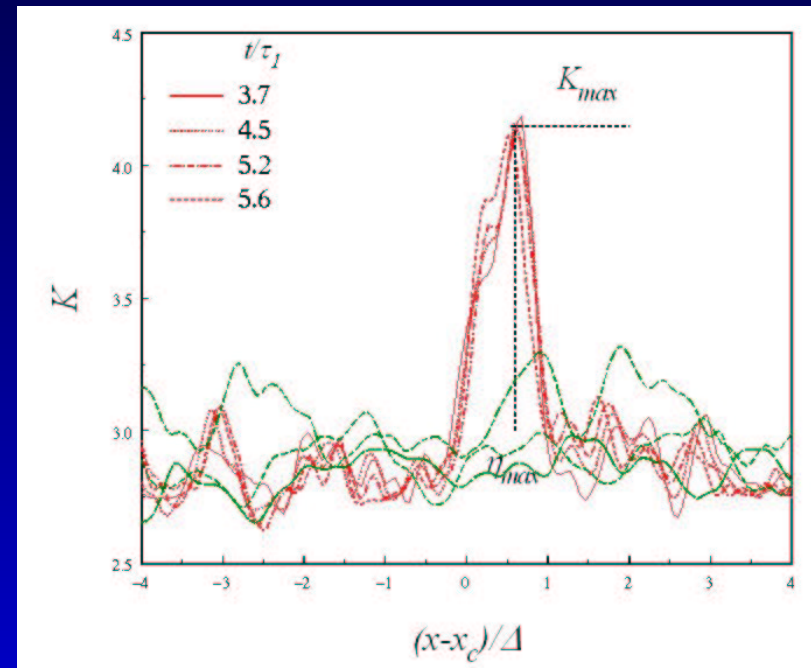
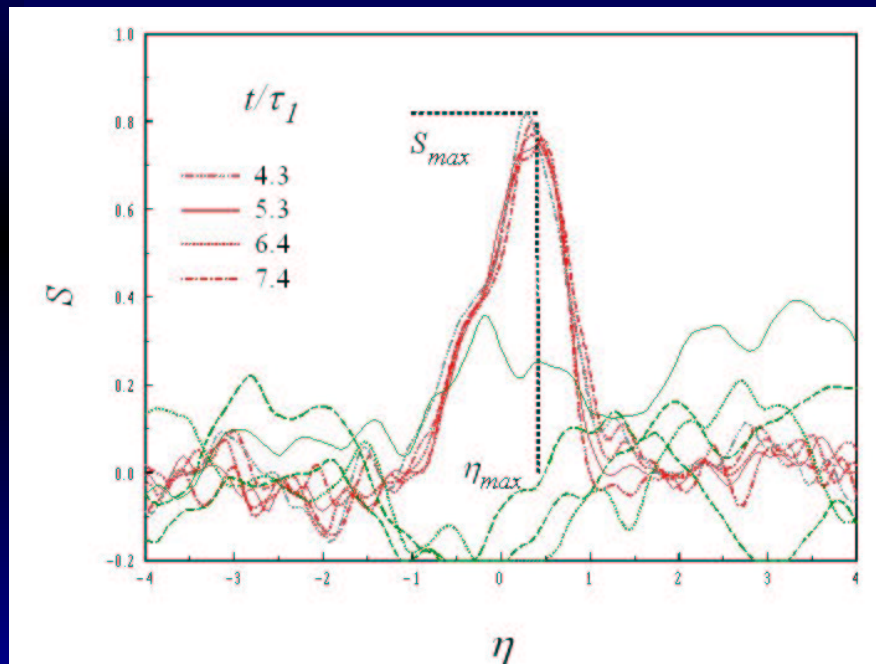
Velocity moments, large scale anisotropy



Velocity moments, large scale intermittency

$$Re_\lambda = 45, E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$

$$S = \overline{u^3}/\overline{u^2}^{3/2} \quad S = \overline{v^3}/\overline{v^2}^{3/2}, \quad K = \overline{u^4}/\overline{u^2}^2 \quad K = \overline{v^4}/\overline{v^2}^2$$



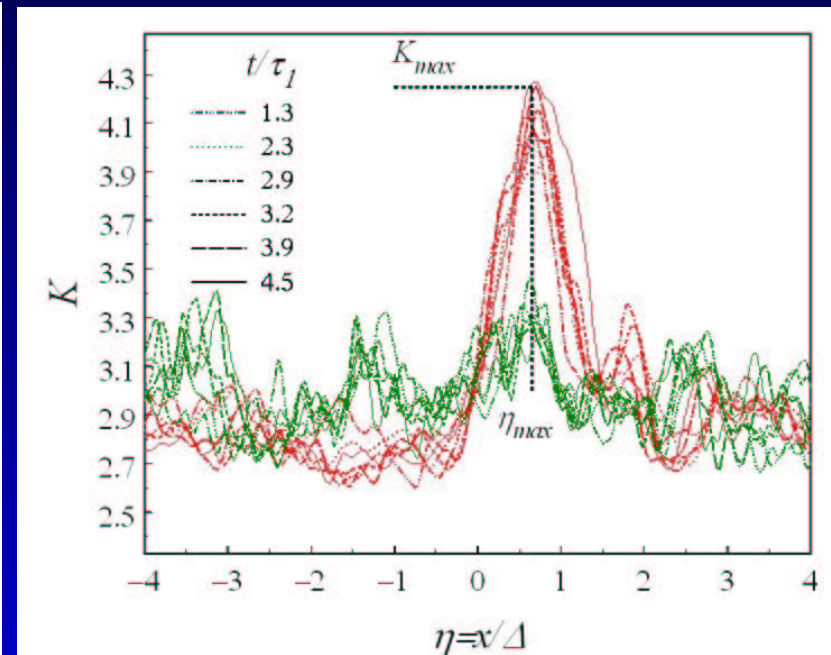
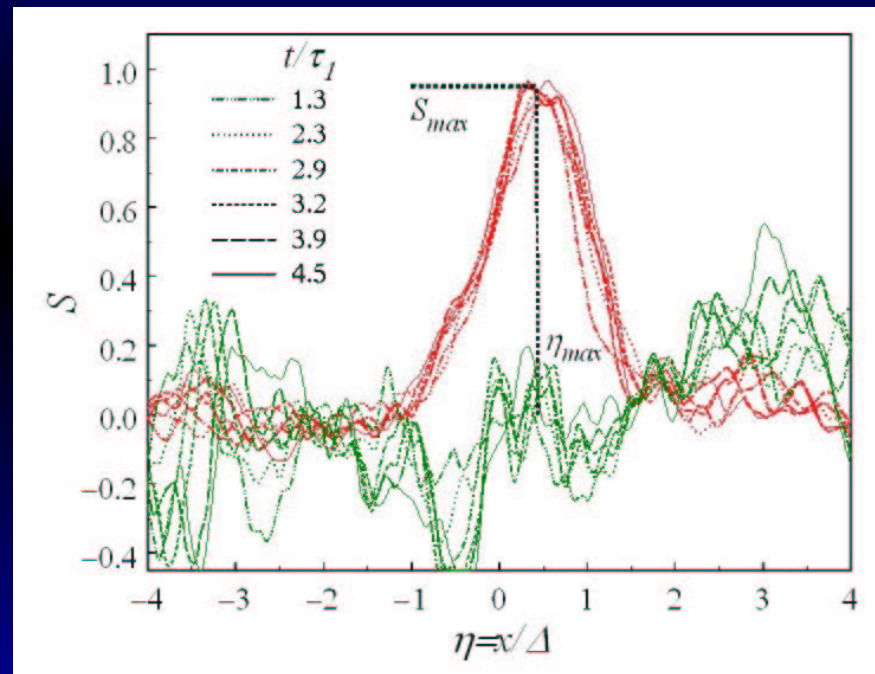
u, x in the mixing direction
 v, y normal to the mixing direction



Velocity moments, large scale intermittency

$$Re_\lambda = 150, E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$

$$S = \overline{u^3}/\overline{u^2}^{3/2} \quad S = \overline{v^3}/\overline{v^2}^{3/2}, \quad K = \overline{u^4}/\overline{u^2}^2 \quad K = \overline{v^4}/\overline{v^2}^2$$

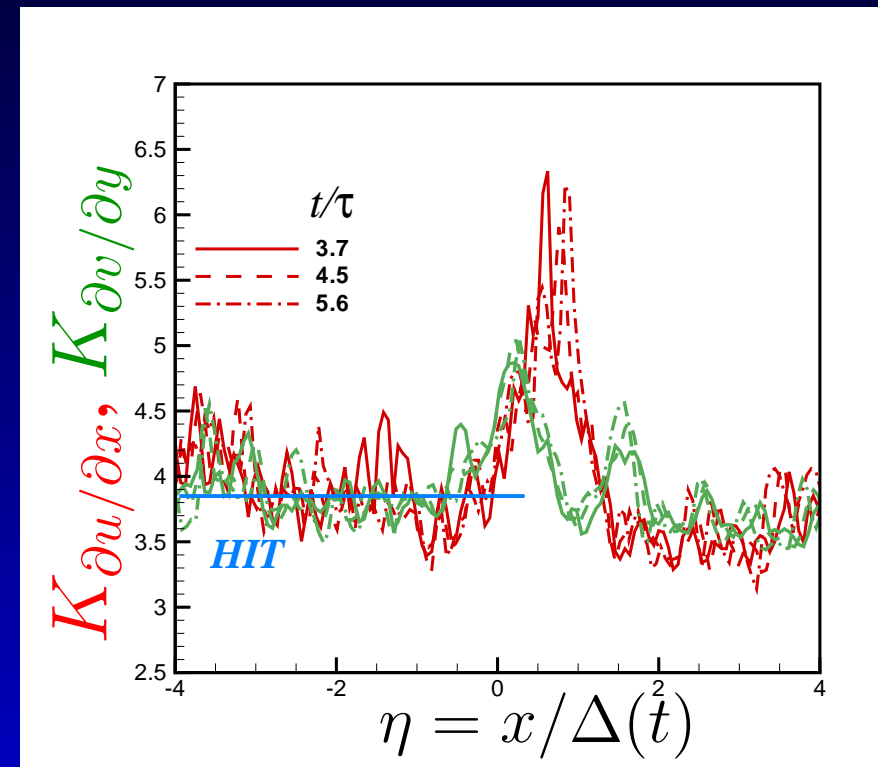
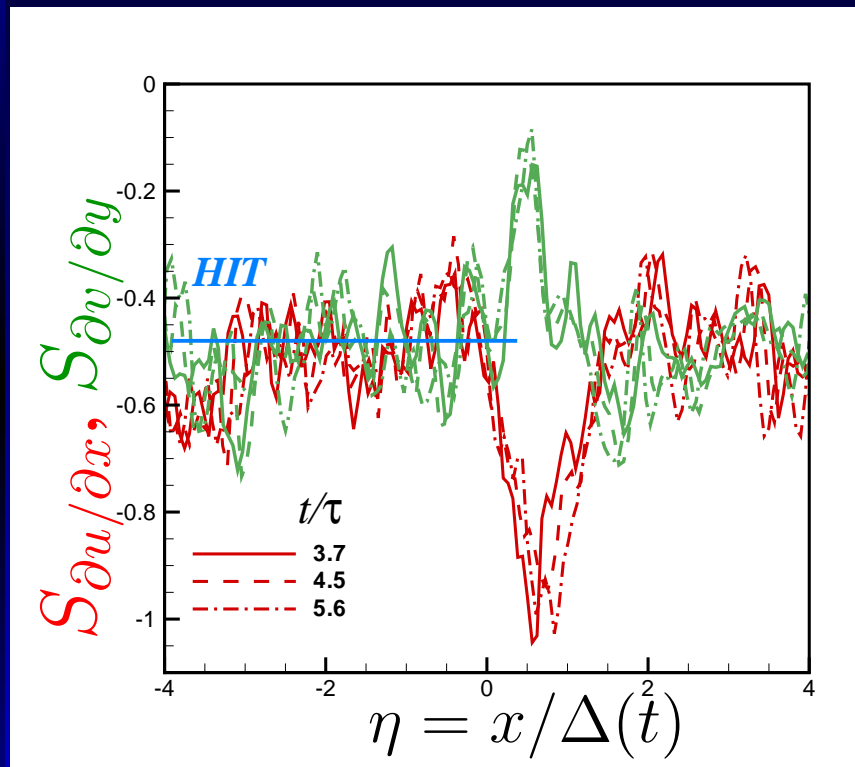


u, x in the mixing direction
 v, y normal to the mixing direction



Small scale intermittency

$$Re_\lambda = 45, E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$

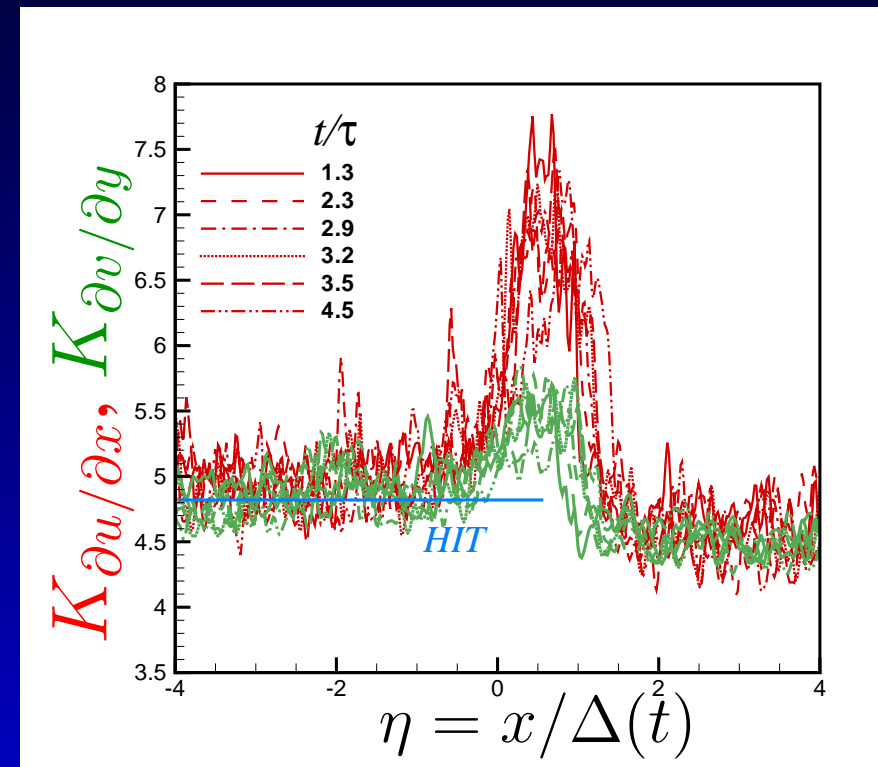
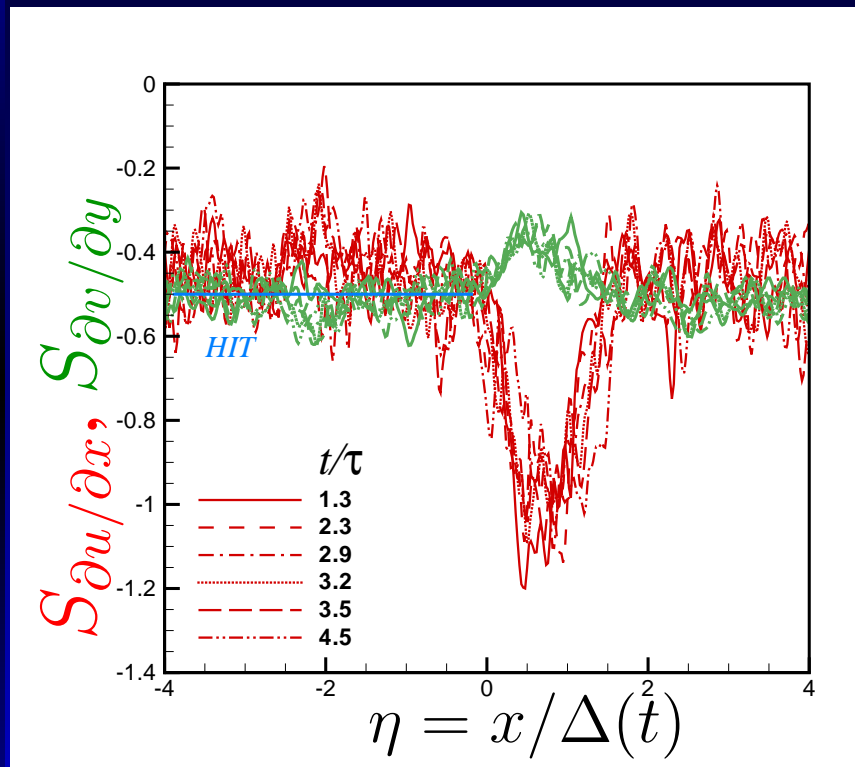


η is the dimensionless coordinate along the mixing
 $\Delta(t)$ is the mixing half-width



Small scale intermittency

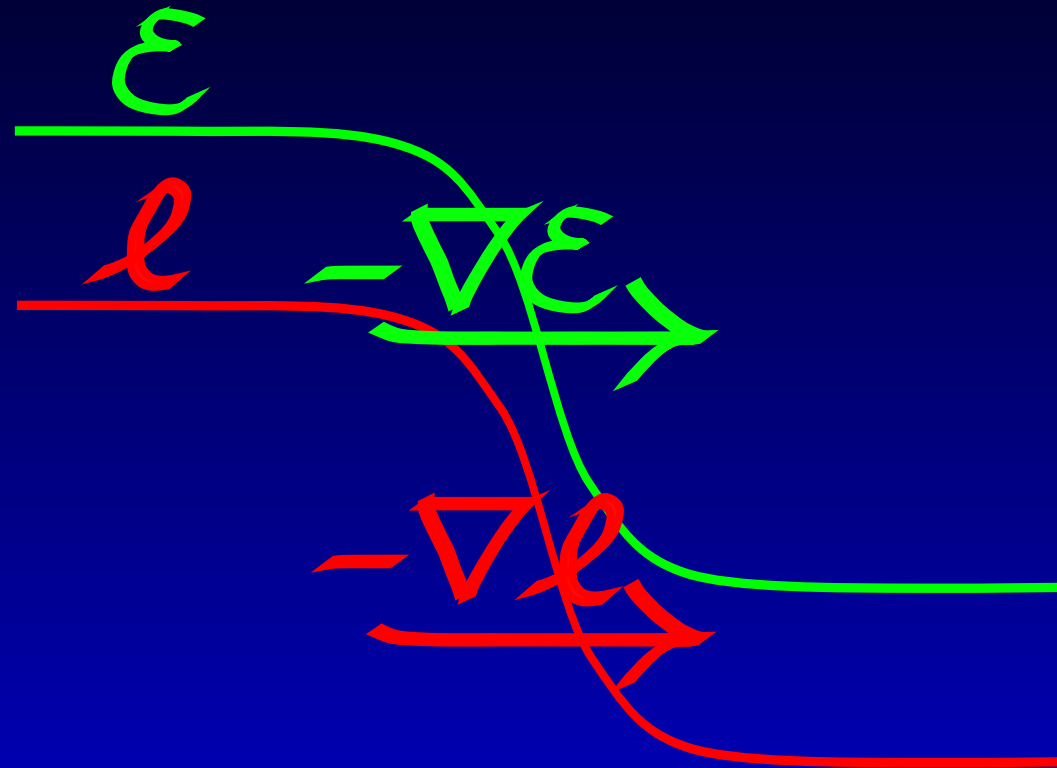
$$Re_\lambda = 150, E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$



η is the dimensionless coordinate along the mixing
 $\Delta(t)$ is the mixing half-width



2-Concurrent gradients

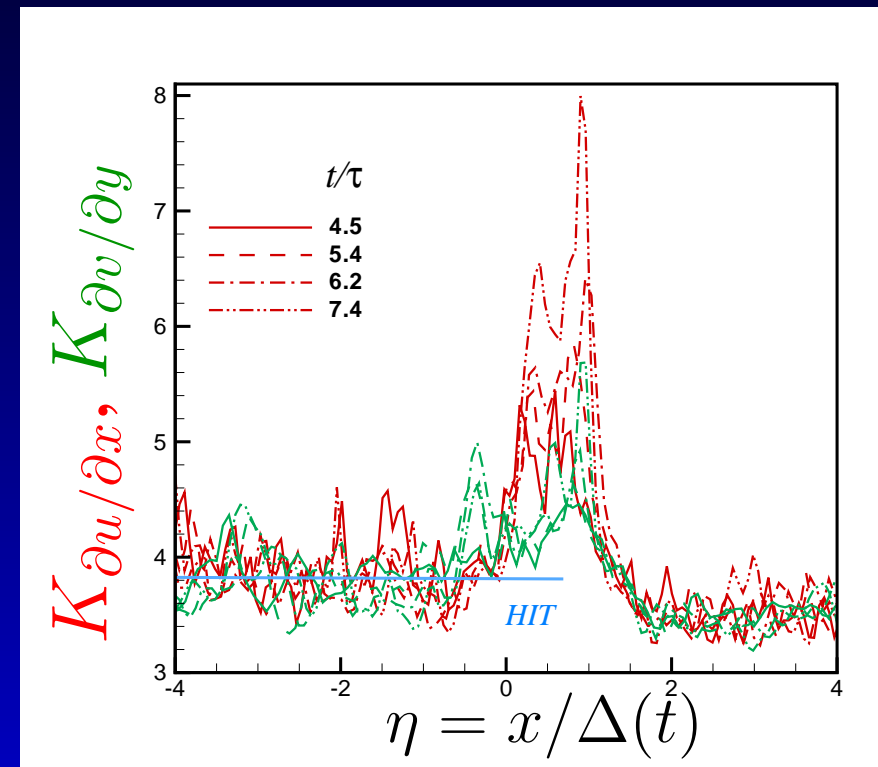
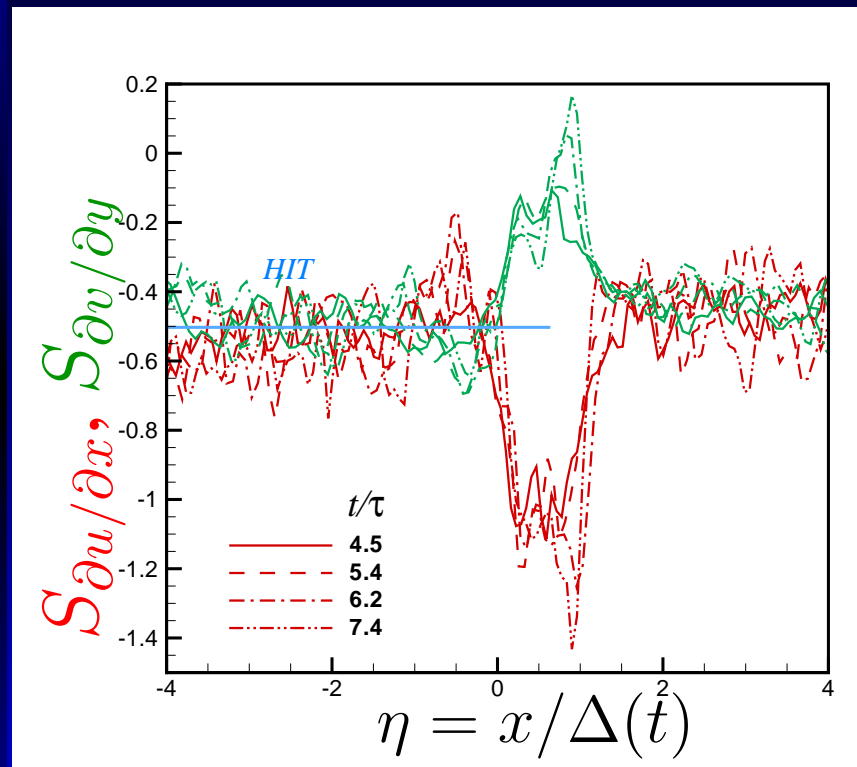


Concurrent gradients of energy and scale



Small scale intermittency

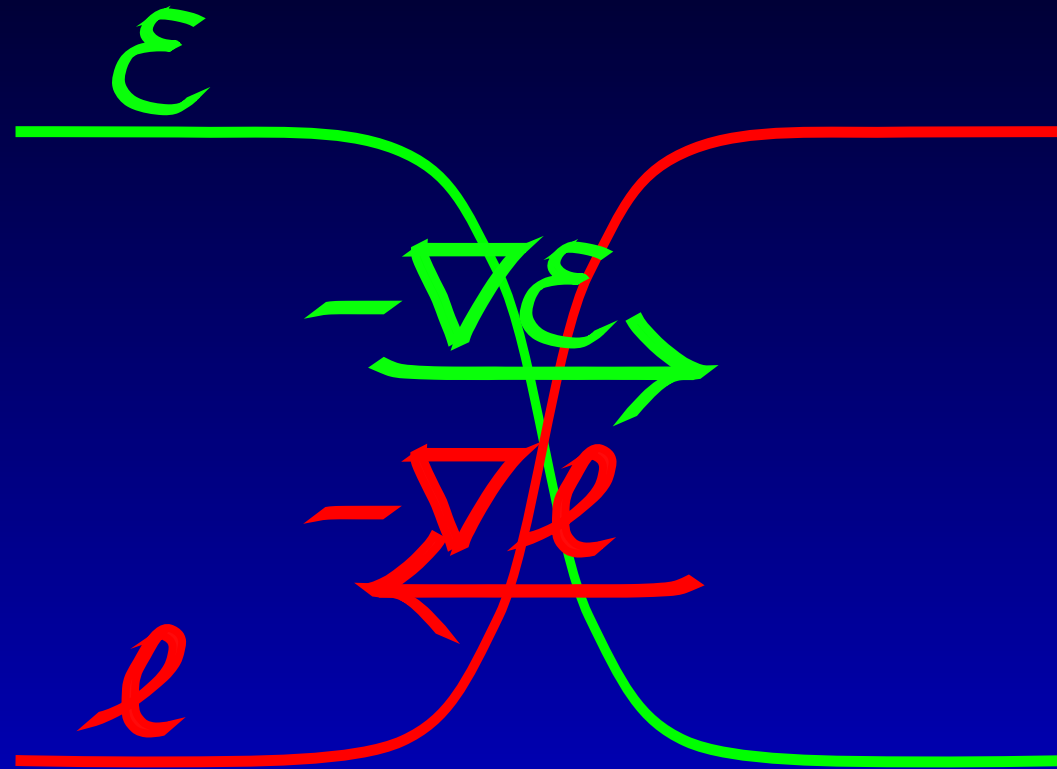
$$Re_\lambda = 45, E_1/E_2 = 6.7, \ell_1/\ell_2 = 2.1$$



η is the dimensionless coordinate along the mixing
 $\Delta(t)$ is the mixing half-width



3-Opposite gradients

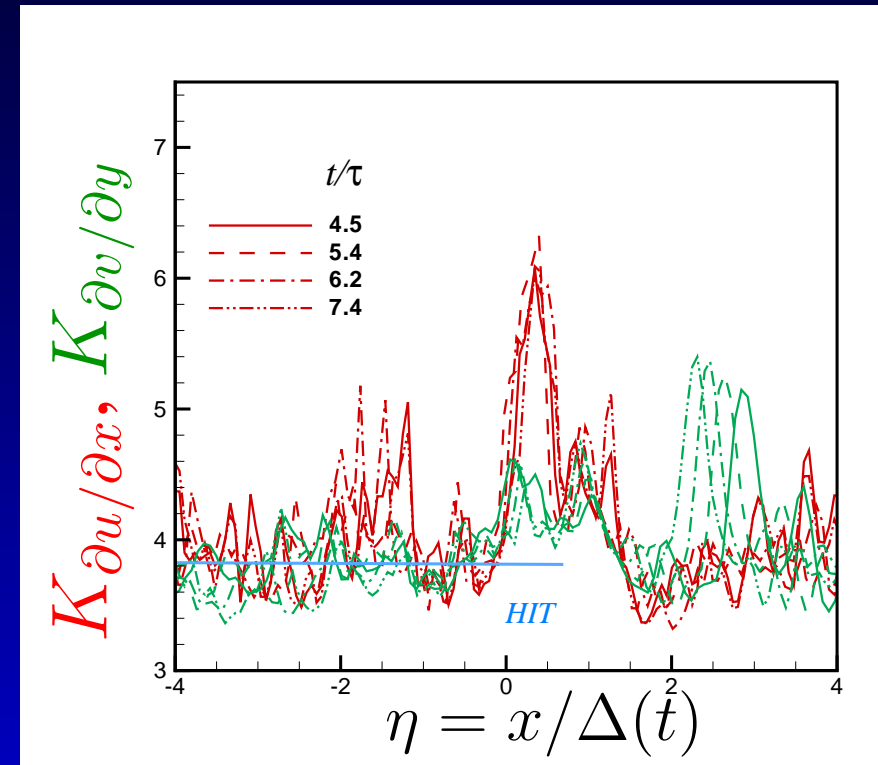
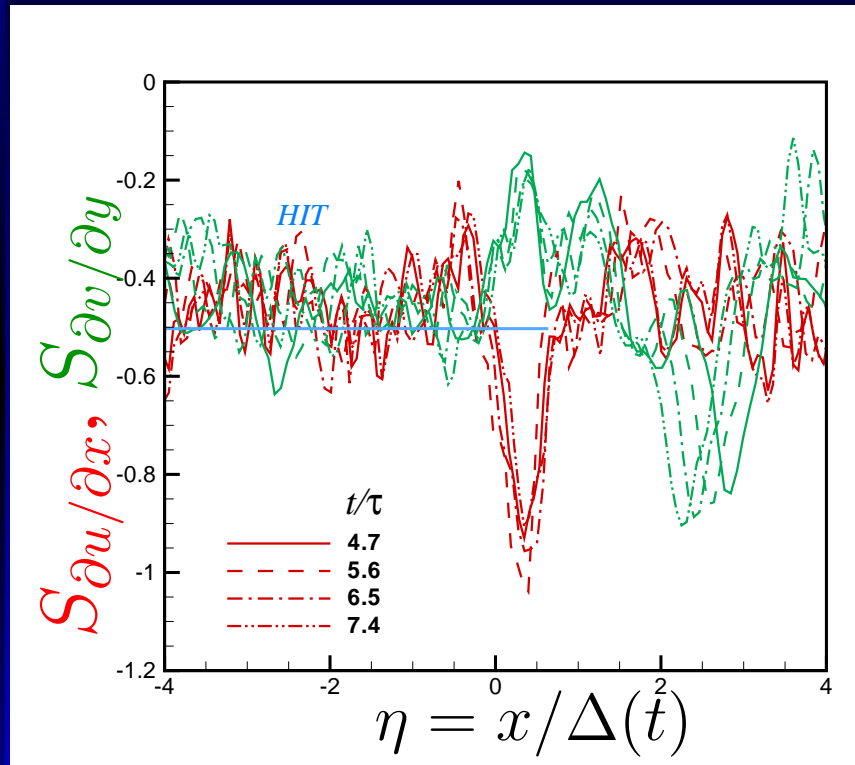


Opposite gradients of energy and integral scale



Small scale intermittency: higher moments

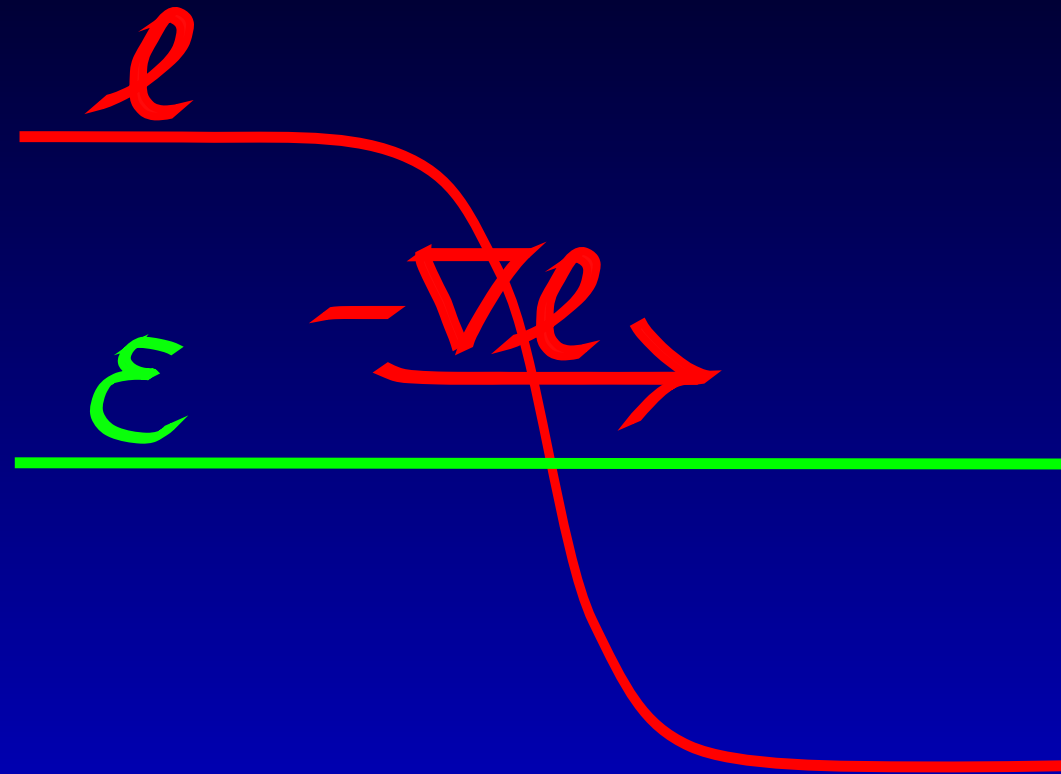
$$Re_\lambda = 45, E_1/E_2 = 6.5, \ell_1/\ell_2 = 0.6$$



η is the dimensionless coordinate along the mixing
 $\Delta(t)$ is the mixing half-width



4-Gradient of integral scale

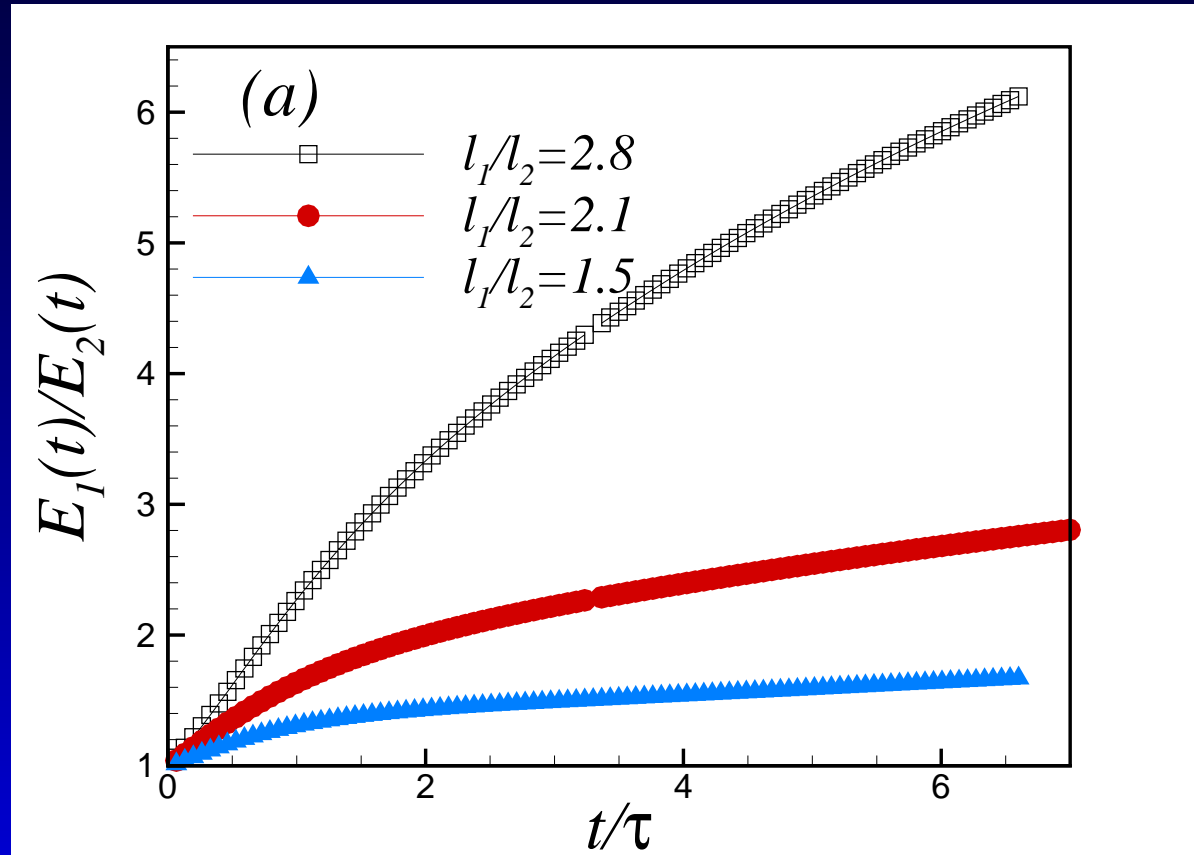


Gradient of integral scale, initially uniform energy



Energy ratio

Different decay rates \Rightarrow kinetic energy does not remain constant:

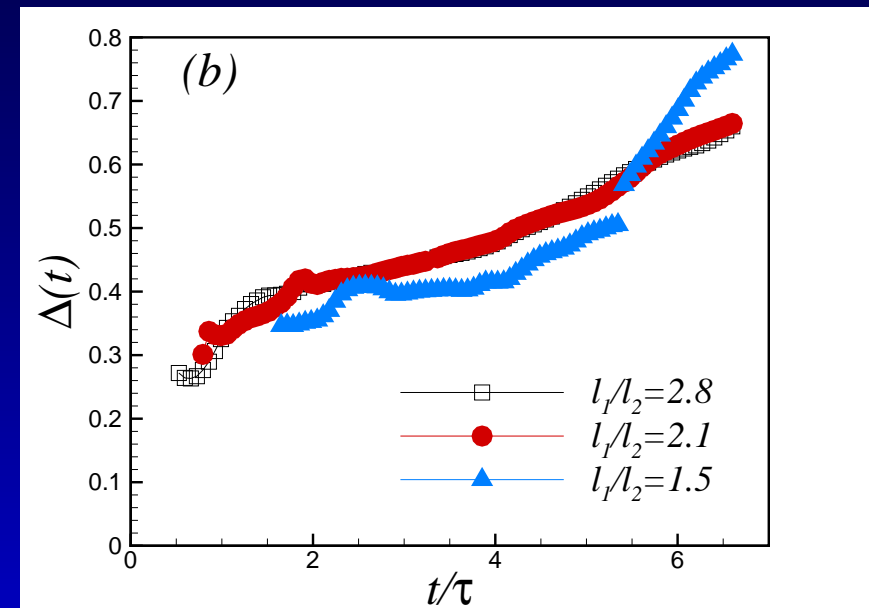
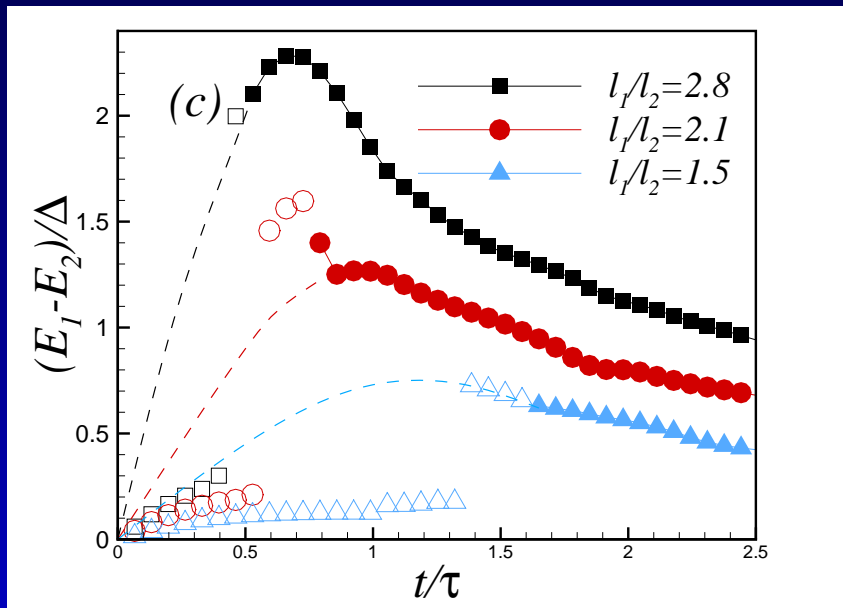


\Rightarrow a concurrent energy gradient is generated



Kinetic energy gradient

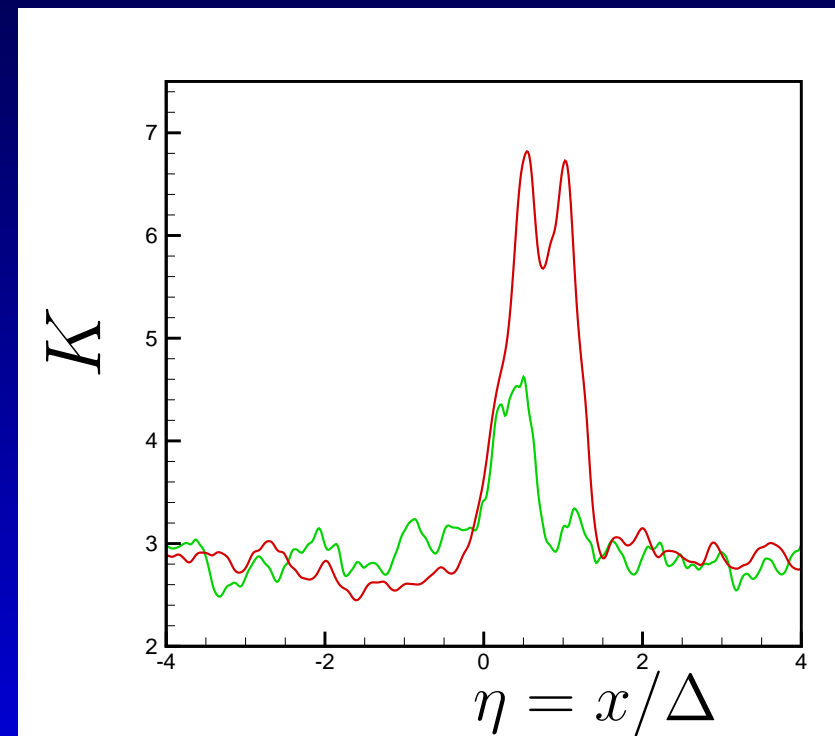
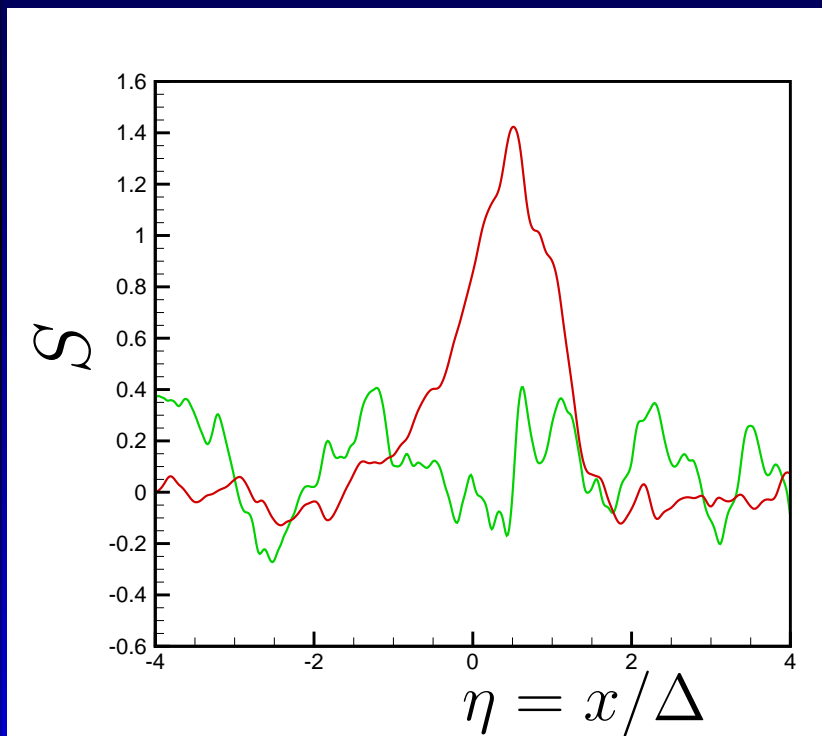
Kinetic energy gradient and mixing layer thickness



Velocity moments, large scale intermittency

$$Re_\lambda = 150, E_1/E_2 = 6.7, \ell_1/\ell_2 = 2.8, t/\tau = 6.8$$

$$S = \overline{u^3}/\overline{u^2}^{3/2} \quad S = \overline{v^3}/\overline{v^2}^{3/2} \quad K = \overline{u^4}/\overline{u^2}^2 \quad K = \overline{v^4}/\overline{v^2}^2$$

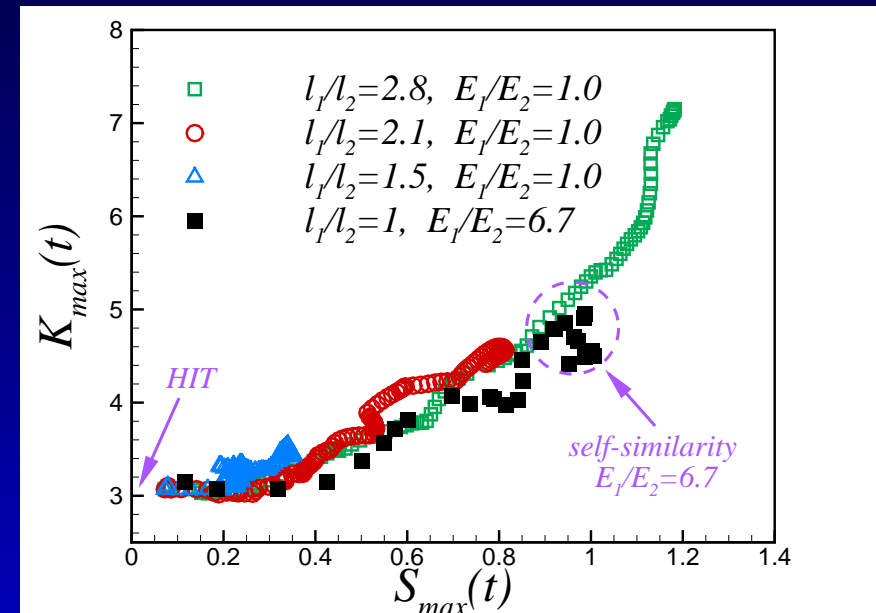
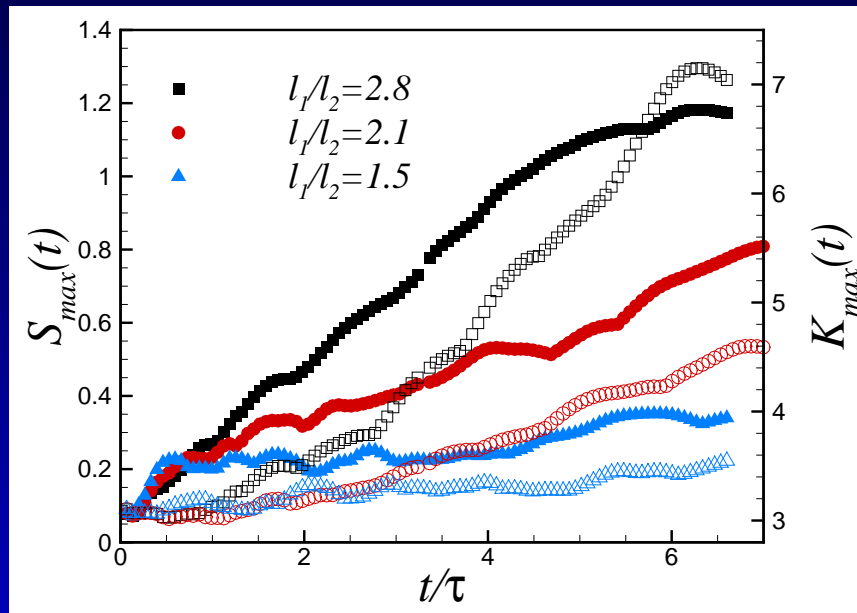


u, x in the mixing direction
 v, y normal to the mixing direction



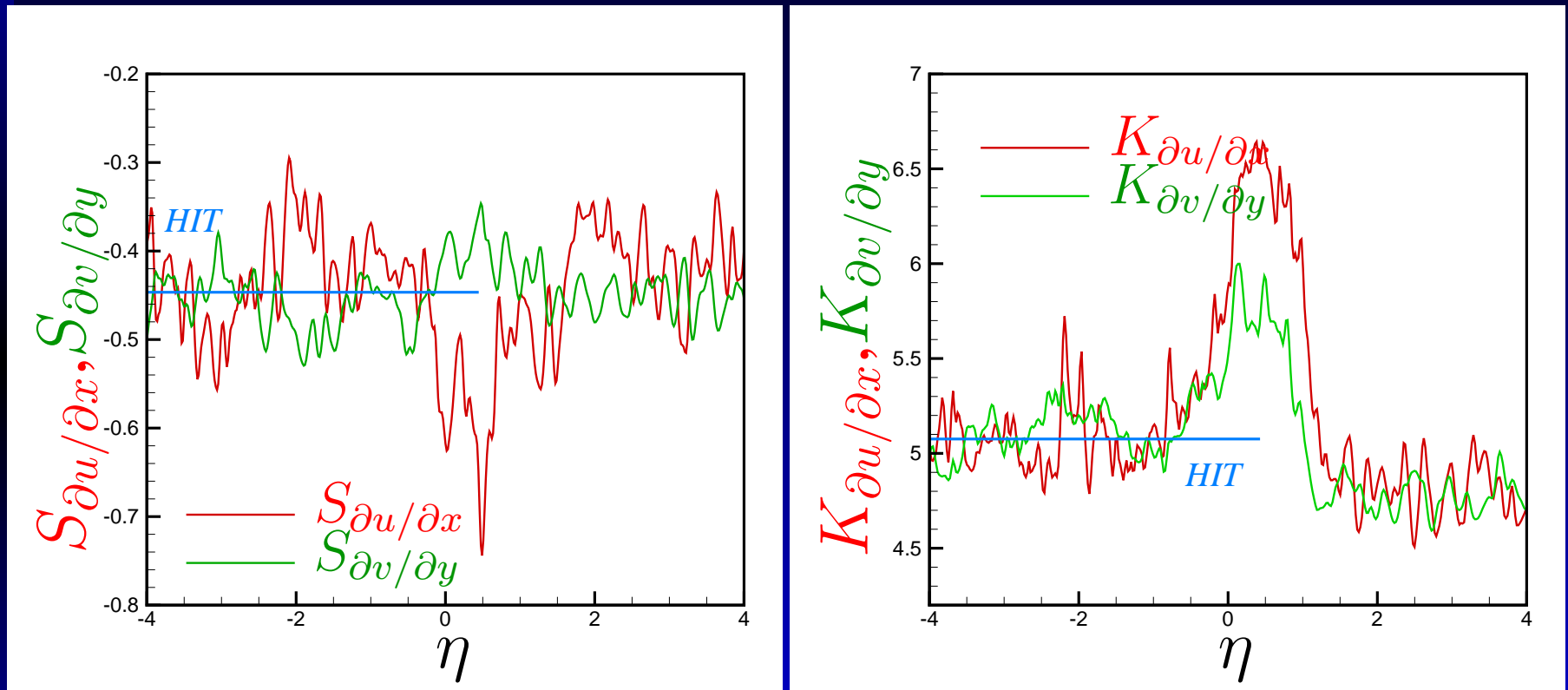
Large scale intermittency

Velocity skewness and kurtosis, component in the inhomogeneous direction: maximum in the mixing layer



Small scale intermittency

$$Re_\lambda = 150, E_1/E_2 = 1, \ell_1/\ell_2 = 2.8, t/\tau = 6.7$$



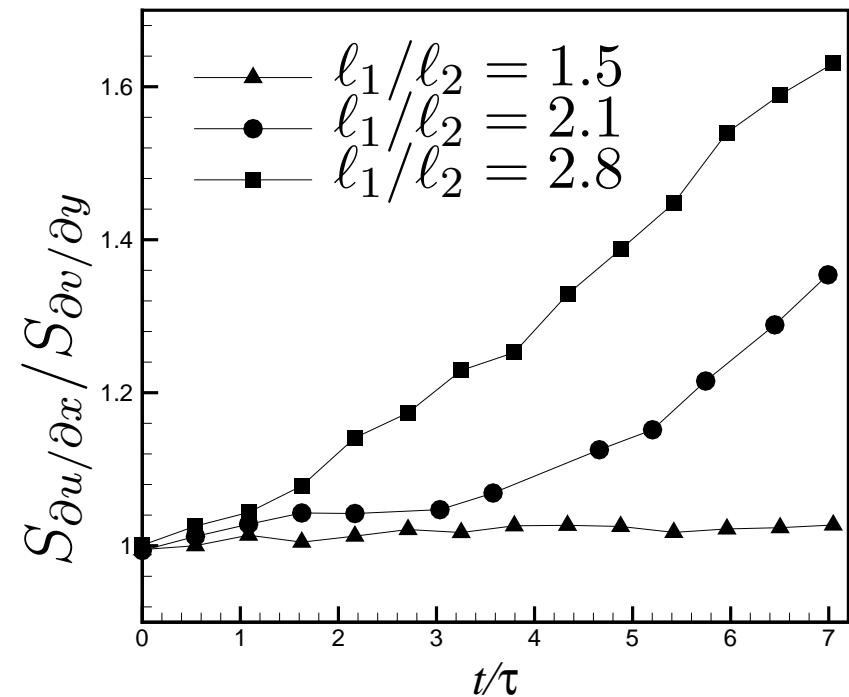
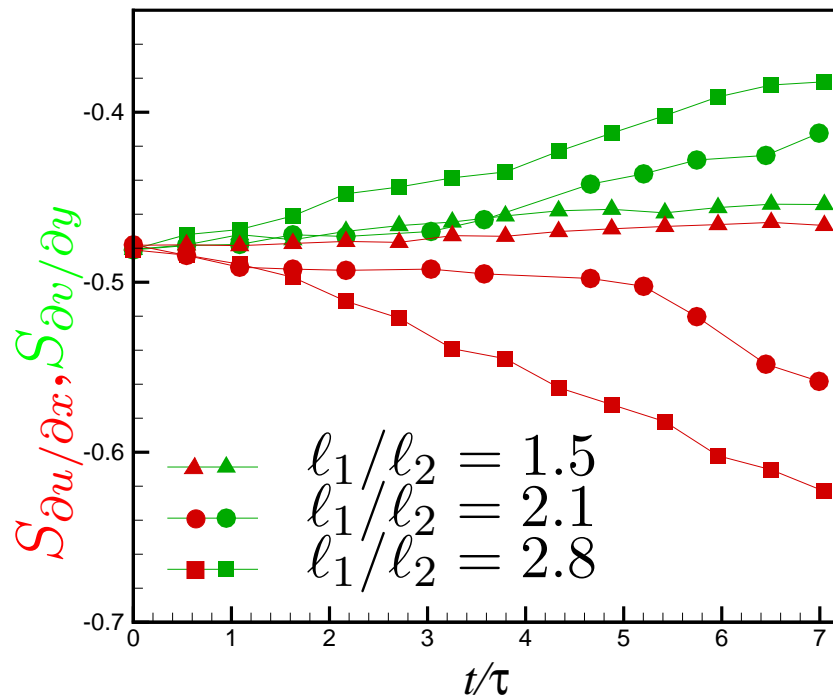
Spatial distribution of longitudinal moments,

$$\eta = x/\Delta,$$



Small scale anisotropy: skewness

$Re_\lambda = 150$, $E_1/E_2 = 1$, $l_1/l_2 = 2.8$:

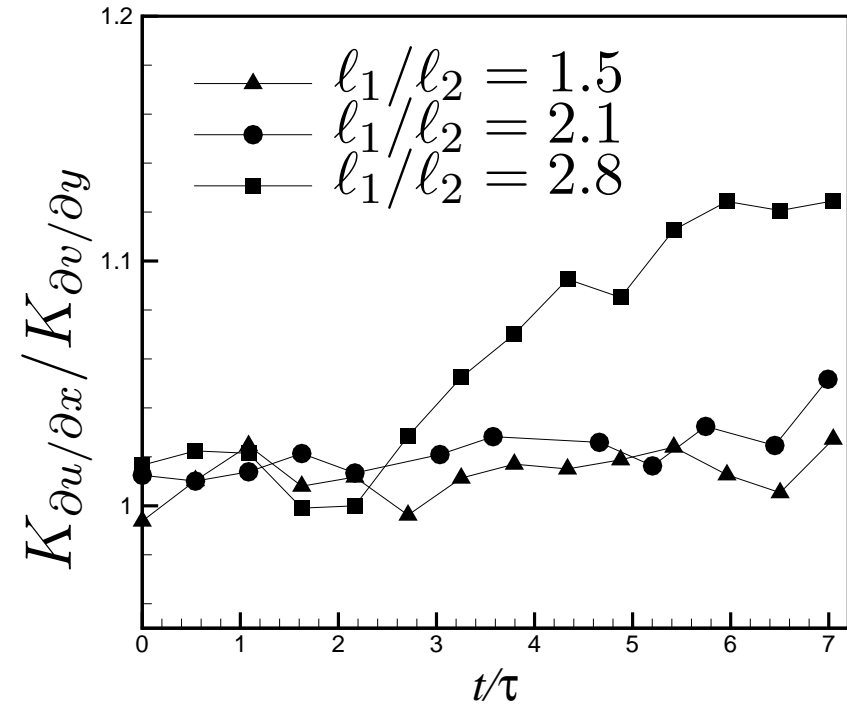
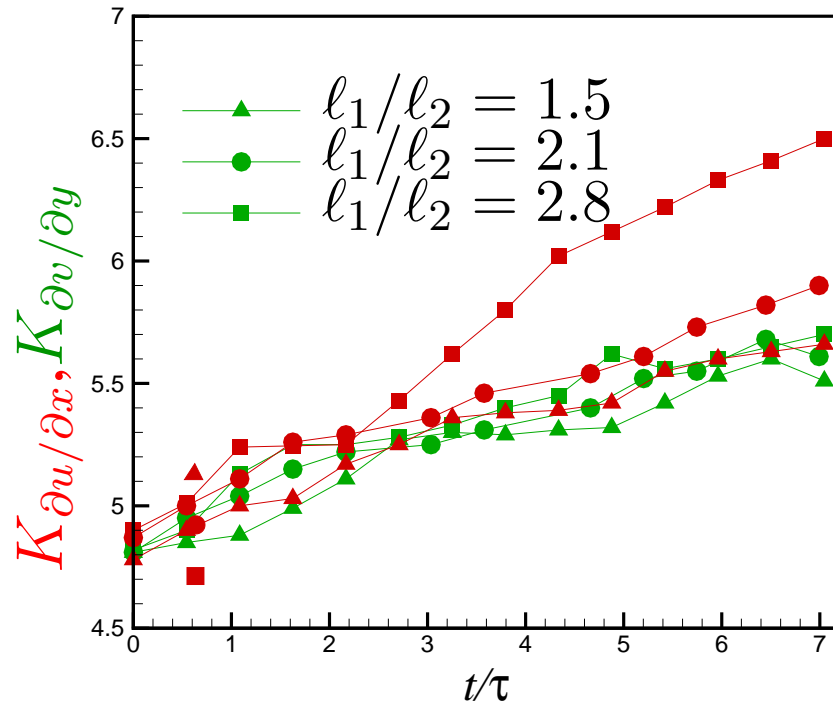


Anisotropy is propagated to small scales.



Small scale anisotropy: kurtosis

$Re_\lambda = 150$, $E_1/E_2 = 1$, $l_1/l_2 = 2.8$:

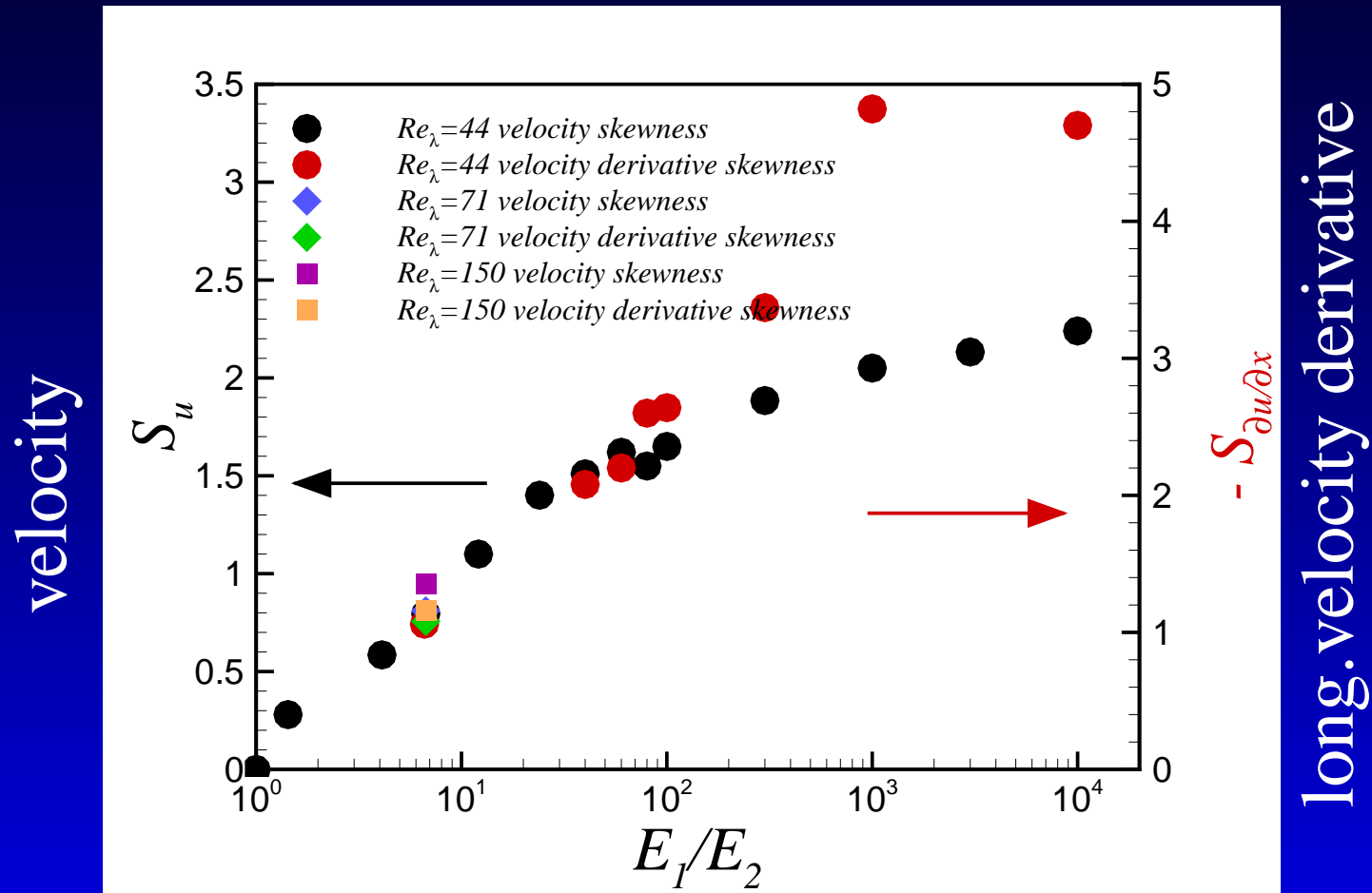


Anisotropy is propagated to small scales.



Asymptote for $E_1/E_2 \rightarrow +\infty$

Skewness:

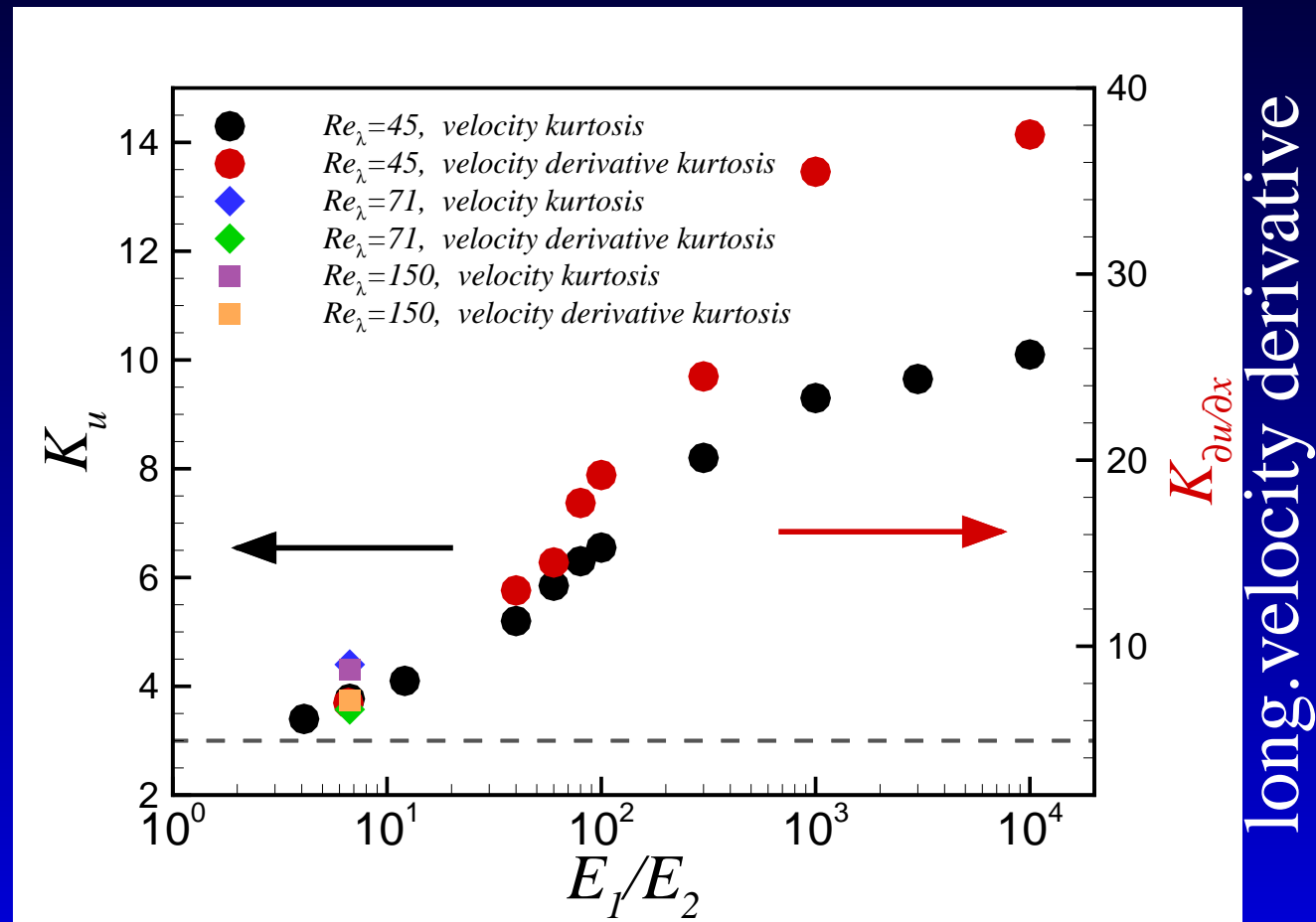


u, x in the mixing direction



Asymptote for $E_1/E_2 \rightarrow +\infty$

Kurtosis:

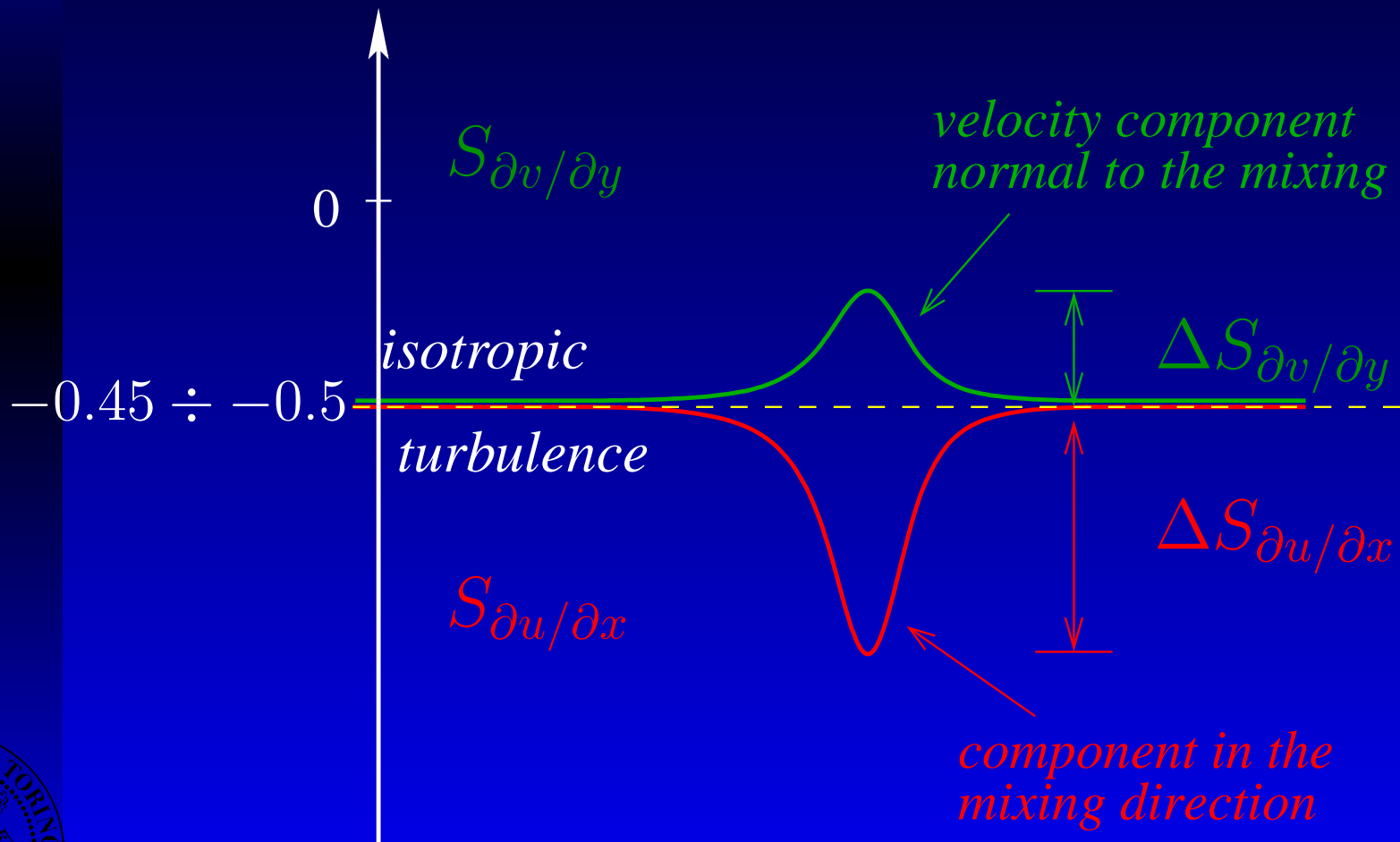


u, x in the mixing direction



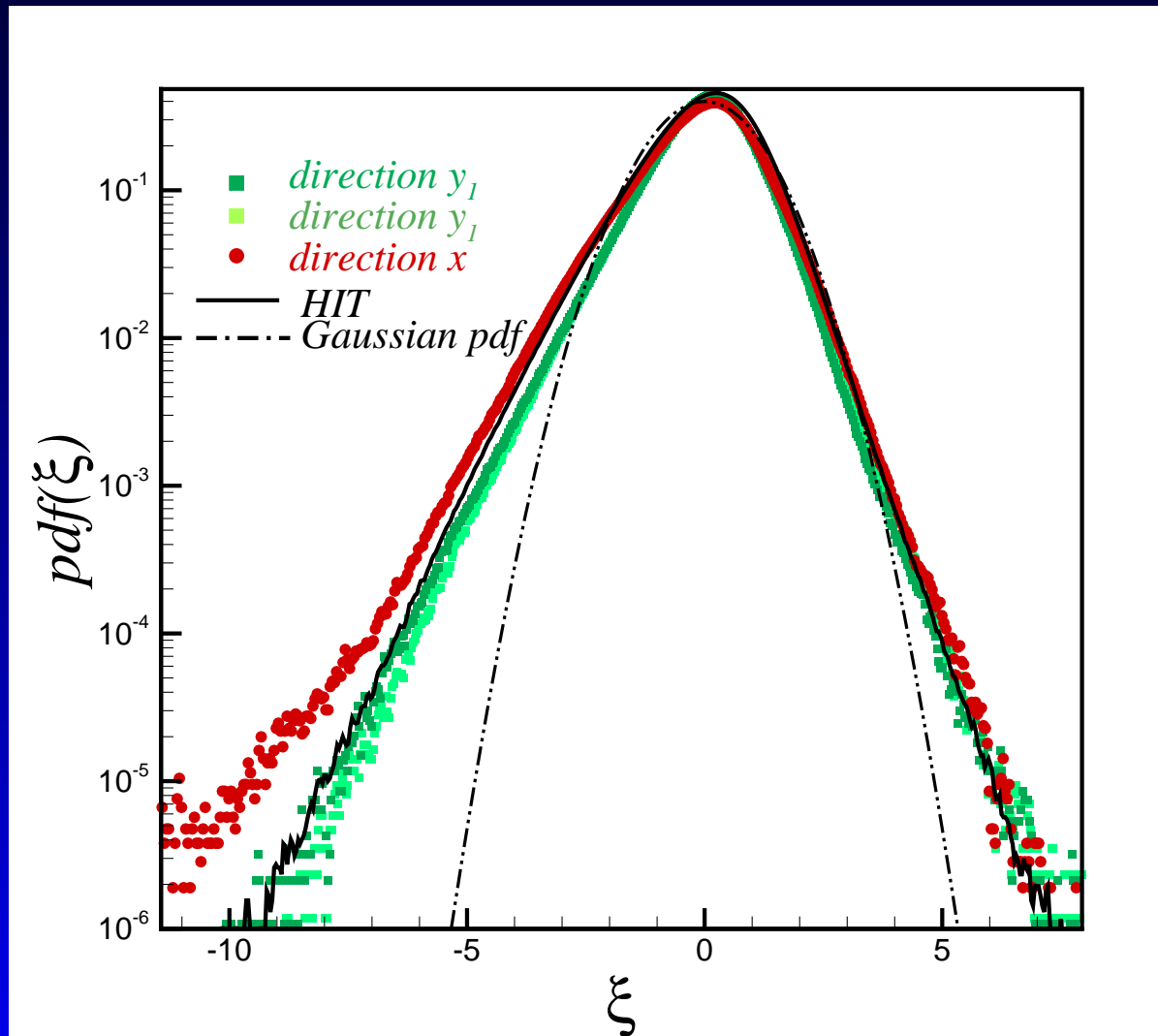
Longitudinal derivatives

Scheme of the general behaviour for the longitudinal skewness



Probability density function

$Re_\lambda = 150, E_1/E_2 = 6.7, t/\tau = 4.0:$



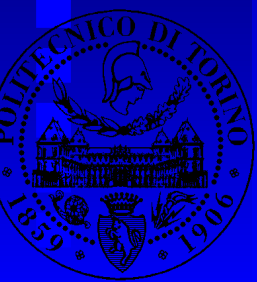
$$\xi = \frac{\partial u_i}{\partial x_i} / \left(\frac{\partial u_i}{\partial x_i} \right)^2 \frac{1}{2}$$

$$i = y_1, y_2, x$$



Large scales: main features of velocity statistics

- *HIGH INTERMITTENCY* function of:
 - ▶ gradient of turbulent kinetic energy
 - ▶ gradient of integral scale
- *ANISOTROPY* mild on the second order moments
 - high for higher moments (anisotropy ratio equal to 2 for the 3rd and 1.5 for the 4th order moments) slightly increasing with Re



Small scales: main features of velocity derivative statistics

- *HIGH INTERMITTENCY* function of:
 - ▶ gradient of turbulent kinetic energy
 - ▶ gradient of integral scale
 - ▶ much more intense than that of the large scales
- *ANISOTROPY* mild on the second order moments
 - high for higher moments (anisotropy ratio up to 10 for the 3rd order moment and 2 for the 4th moment) slightly decreasing with Re



Conclusions

Simulations of a flow with an homogenous energy and an integral scale gradient show:

- an integral scale inhomogeneity generates an energy gradient
- the decay exponent of turbulent flow with the same initial energy depends on their integral scale \Rightarrow smaller the scale, faster the decay.
- intermittency generated in the mixing layer can be higher than generated by an energy gradient and a uniform scale
- anisotropy and intermittency spread to small scales.



Decay exponent of large and small scales in isotropic turbulence

Euromech Colloquium 512, Torino, October 2009

Michele Iovieno, Daniela Tordella

Dipartimento di Ingegneria Aeronautica e Spaziale

Politecnico di Torino,

Corso Duca degli Abruzzi 24, 10129 Torino, Italy



Motivation

- Verify the dependence of the decay exponent of homogeneous turbulence from the initial conditions.
- check the role of the small scales during the decay



State of art

Speziale - Bernard 1992, self-preserving decay: all correlations scale with the Taylor microscale:

$$B_{LL}(r) = \overline{u'^2} f\left(\frac{r}{L}\right)$$

$$B_{LLL}(r) = \overline{u'^2}^{\frac{3}{2}} g\left(\frac{r}{L}\right)$$

- $L = \lambda(t)$
- decay exponent asymptotes -1
- all length scales proportional to λ during the decay
- derivative skewness $S = \text{constant}$.



State of art

George 1992: equilibrium hypothesis, relaxed constraint on triple correlations:

$$\partial_t E(k, t) = T(k, t) - \nu k^2 E(k, t)$$

$$E(k, t) \approx \overline{u'^2} \lambda f(k\lambda, *)$$

$$T(k, t) \approx \frac{\nu \overline{u'^2}}{\lambda} g(k\lambda, *)$$

- power-law decay determined by the initial conditions (initial Re_λ)
- $S Re_\lambda = \text{const}$



Experiments

Lavoje et al., *JFM* 2007, grid turbulence

Antonia et al., *J.Turb.* 2003, grid turbulence

Antonia, Orlandi *PoF* 2003, dns

Antonia, Orlandi *JFM* 2004, dns

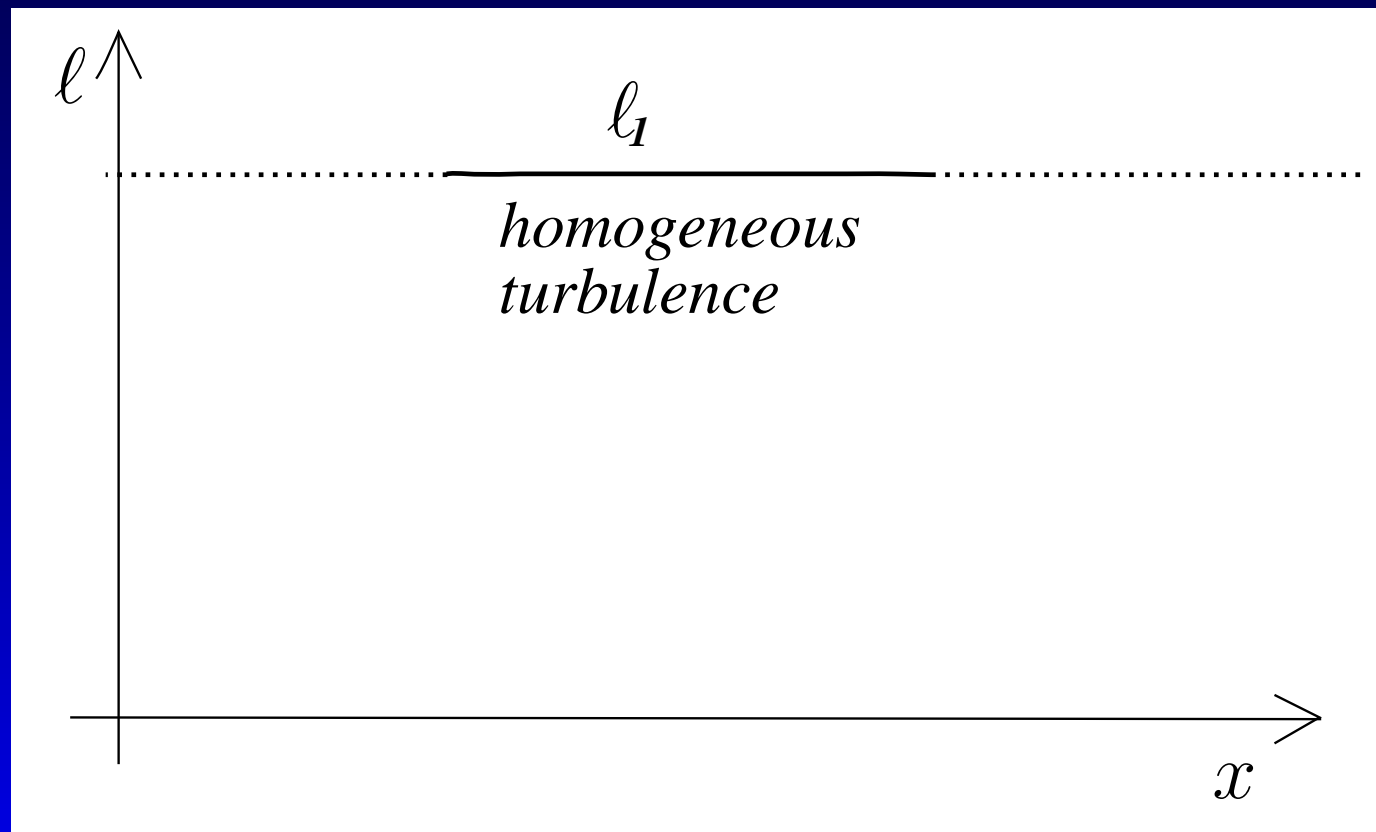
Mansour, Wray, *PoF* 1994, dns

etc.



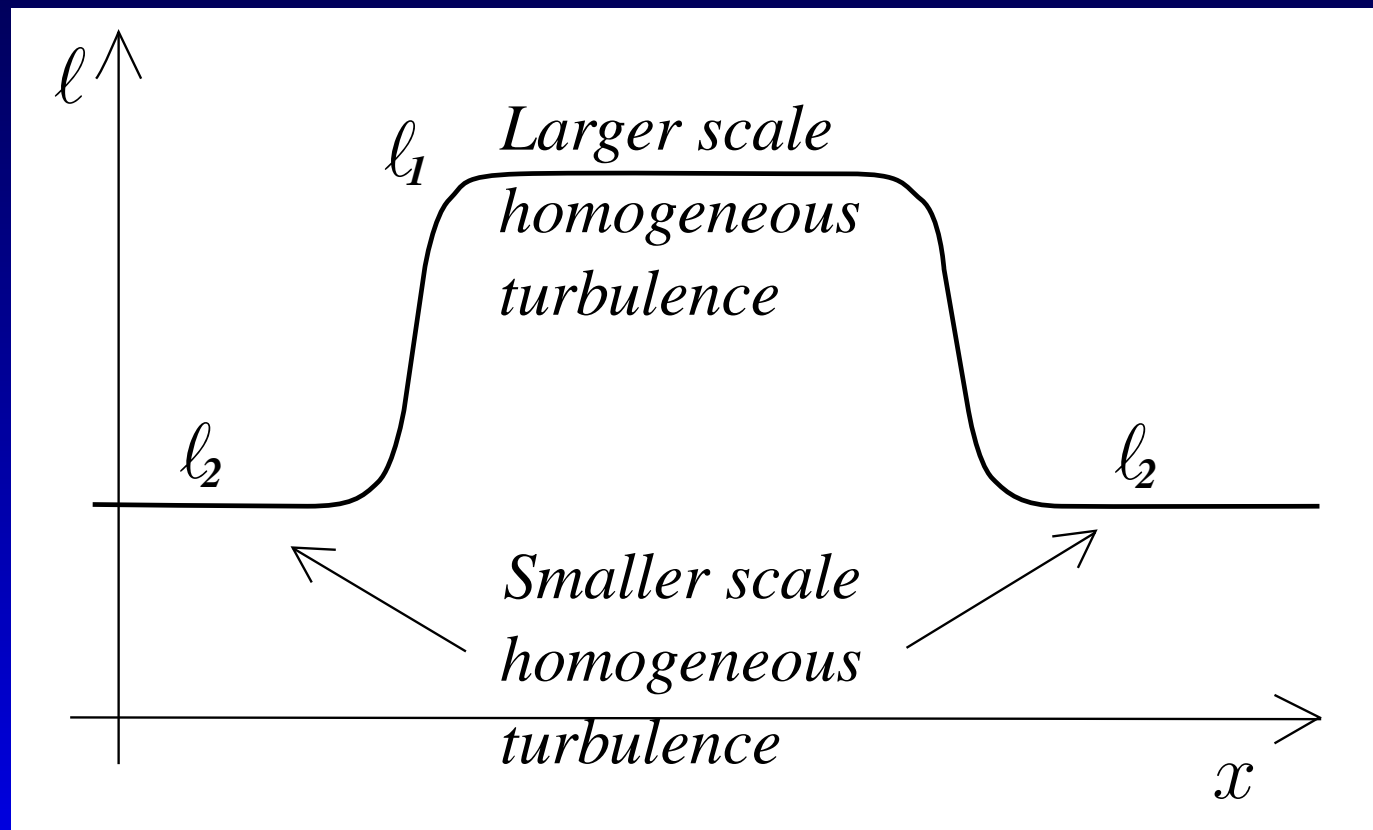
Flow configuration

We follow the time decay of two homogeneous turbulent flows with the same initial kinetic energy but different scales:



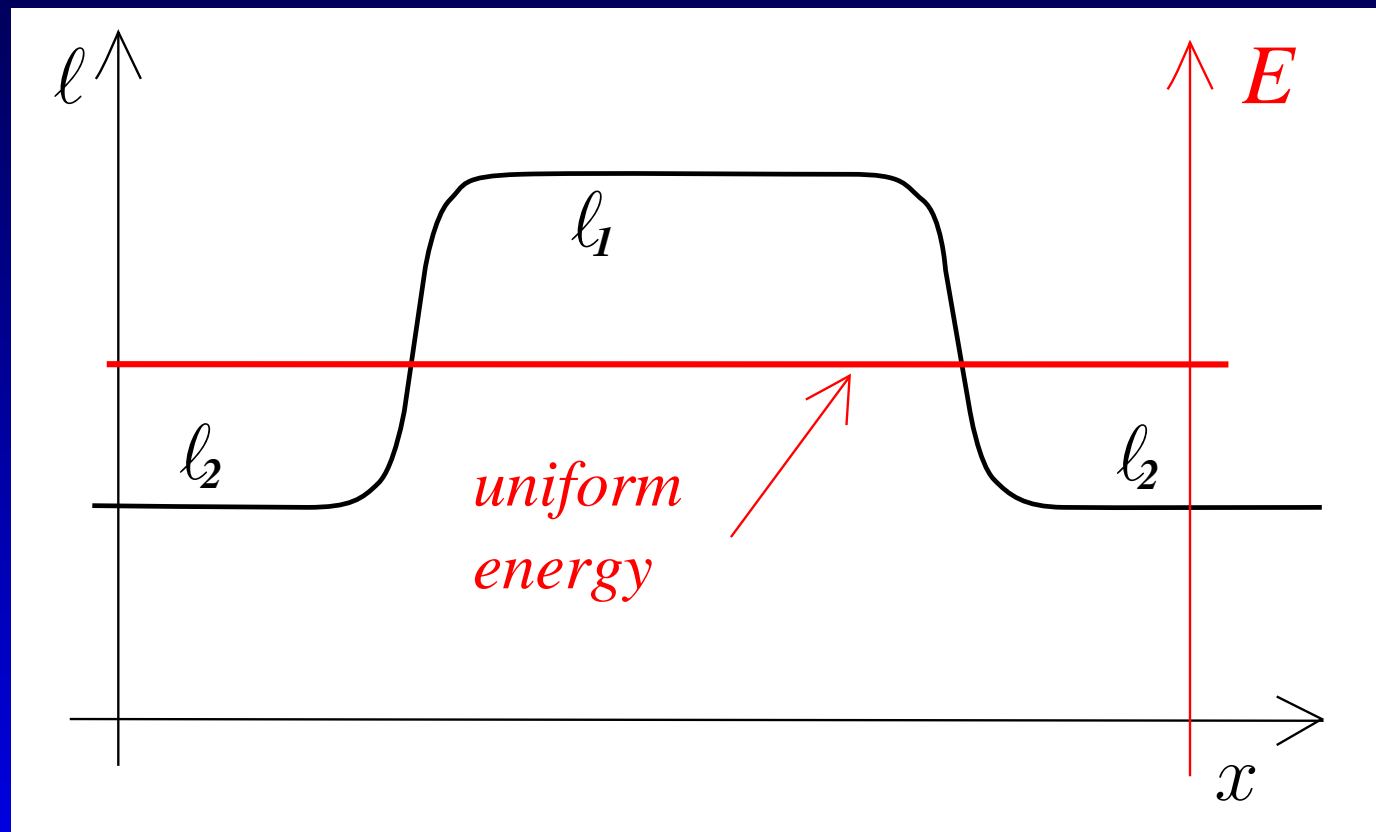
Flow configuration

We follow the time decay of two homogeneous turbulent flows with the same initial kinetic energy but different scales:

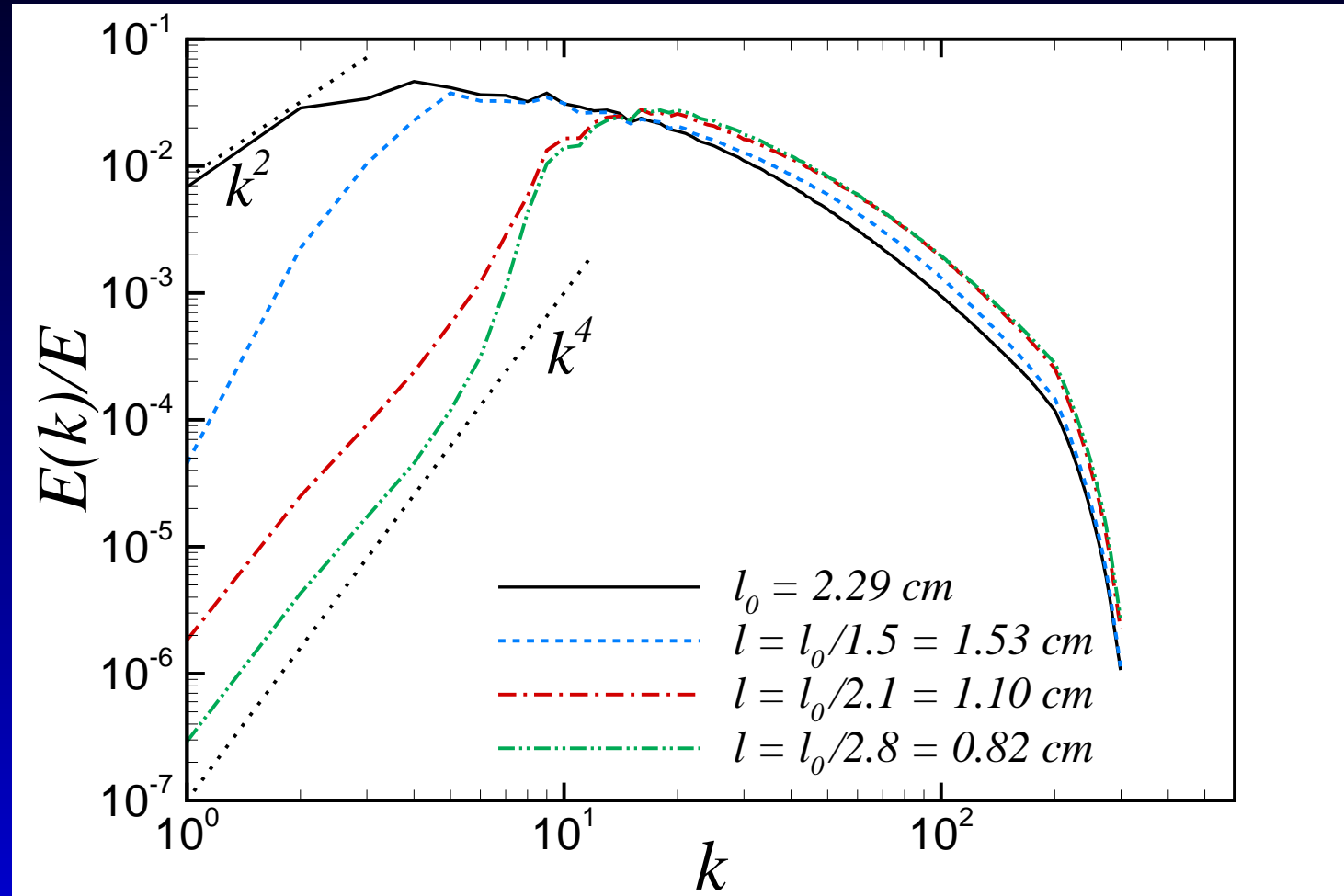


Flow configuration

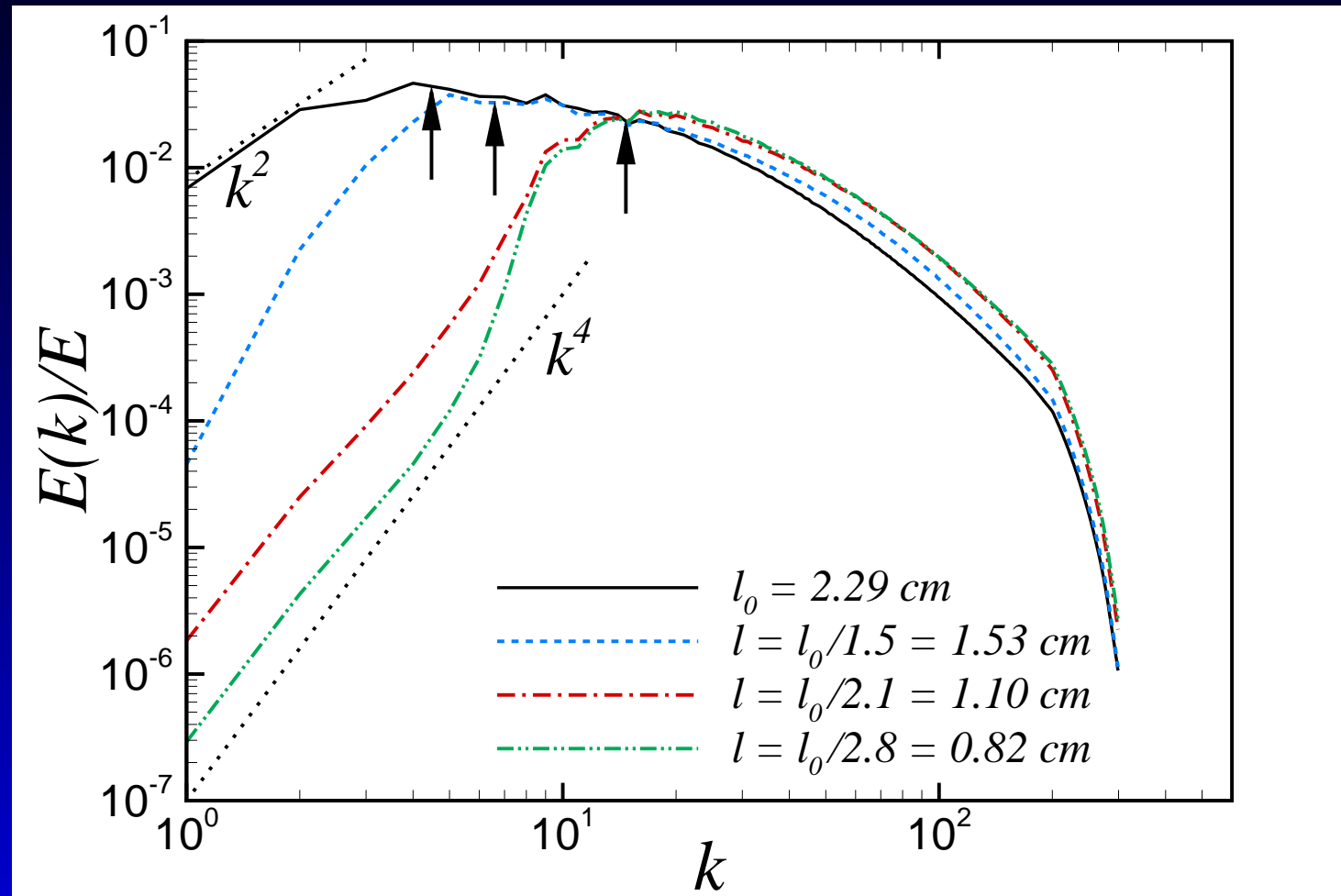
We follow the time decay of two homogeneous turbulent flows with the same initial kinetic energy but different scales:



Initial energy spectra

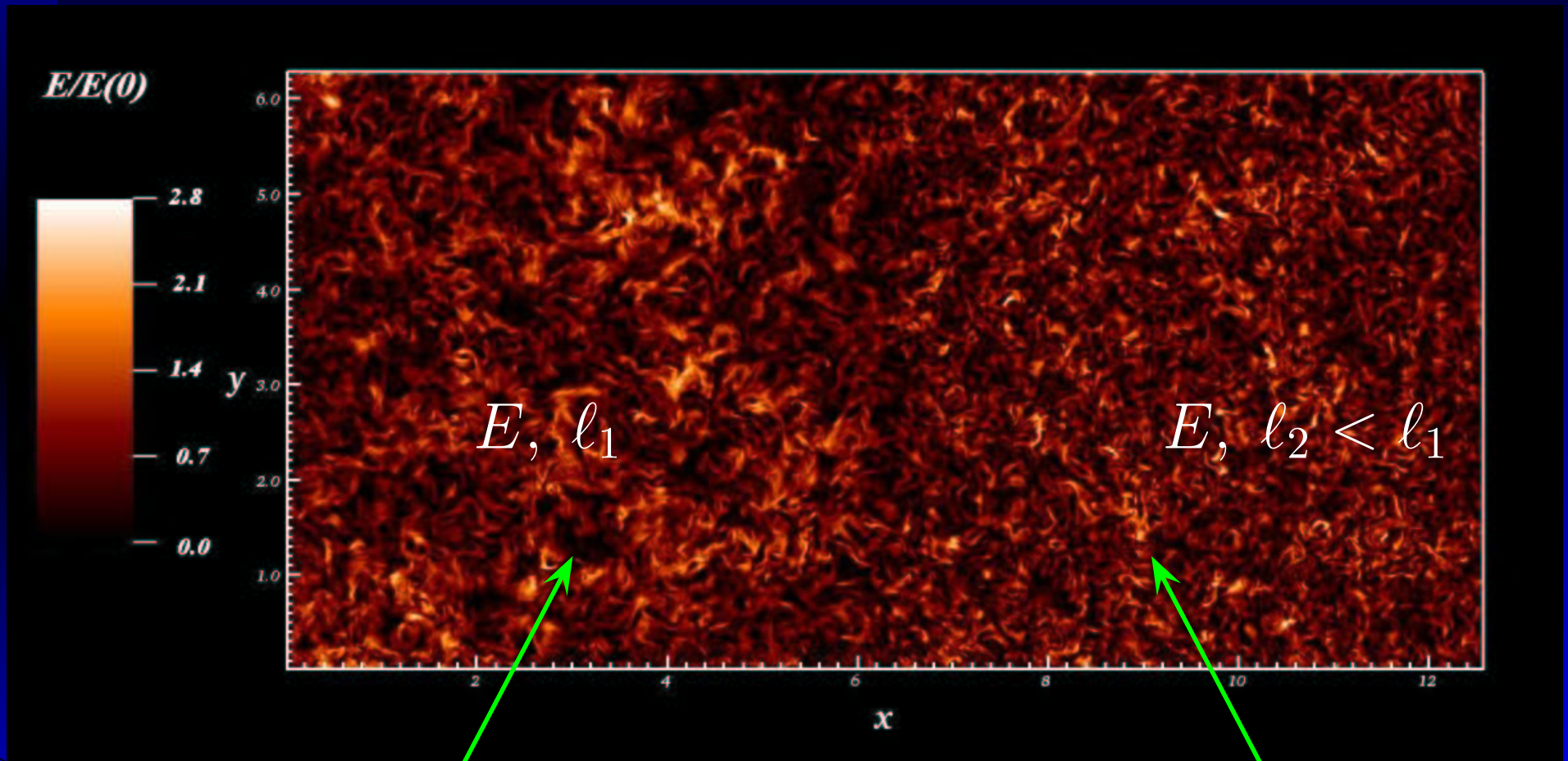


Initial energy spectra



Flow configuration

$$Re_\lambda = 150, E_1 = E_2, \ell_1 > \ell_2, t/\tau = 0$$



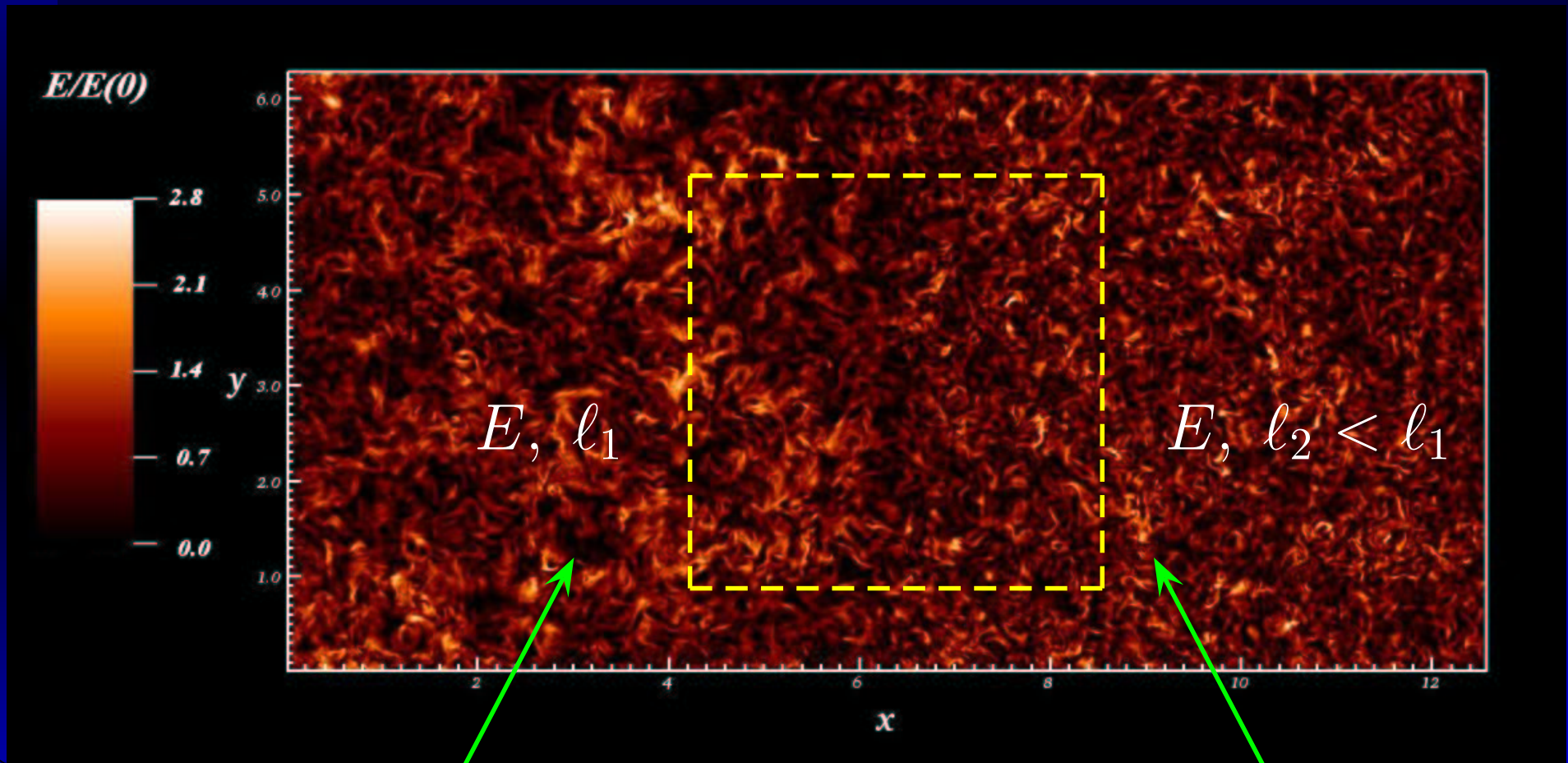
Larger integral scale

Smaller integral scale



Flow configuration

$$Re_\lambda = 150, E_1 = E_2, \ell_1 > \ell_2, t/\tau = 0$$

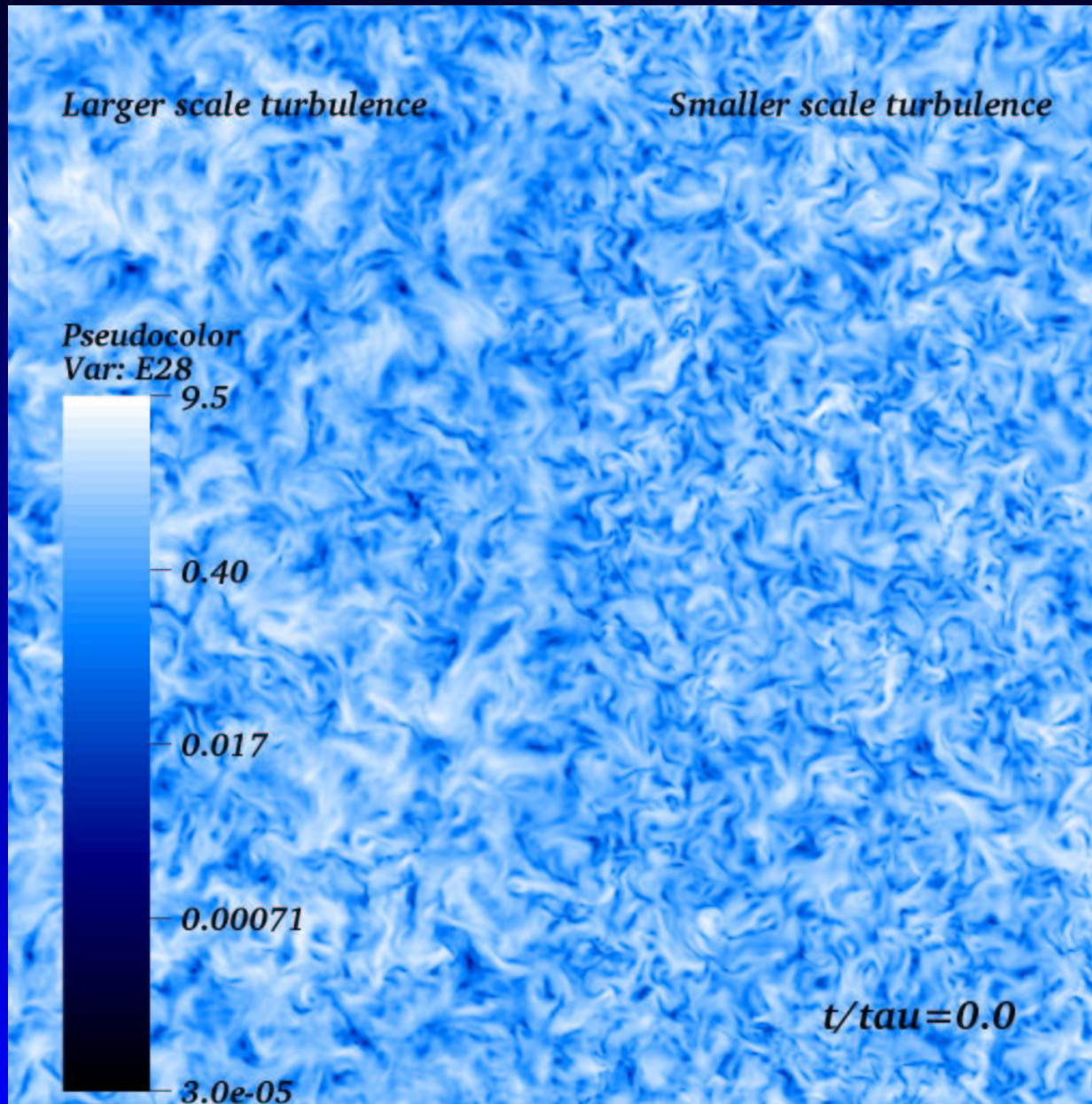


Larger integral scale

Smaller integral scale



Flow configuration



$$Re_\lambda = 150$$

$$E_1 = E_2$$

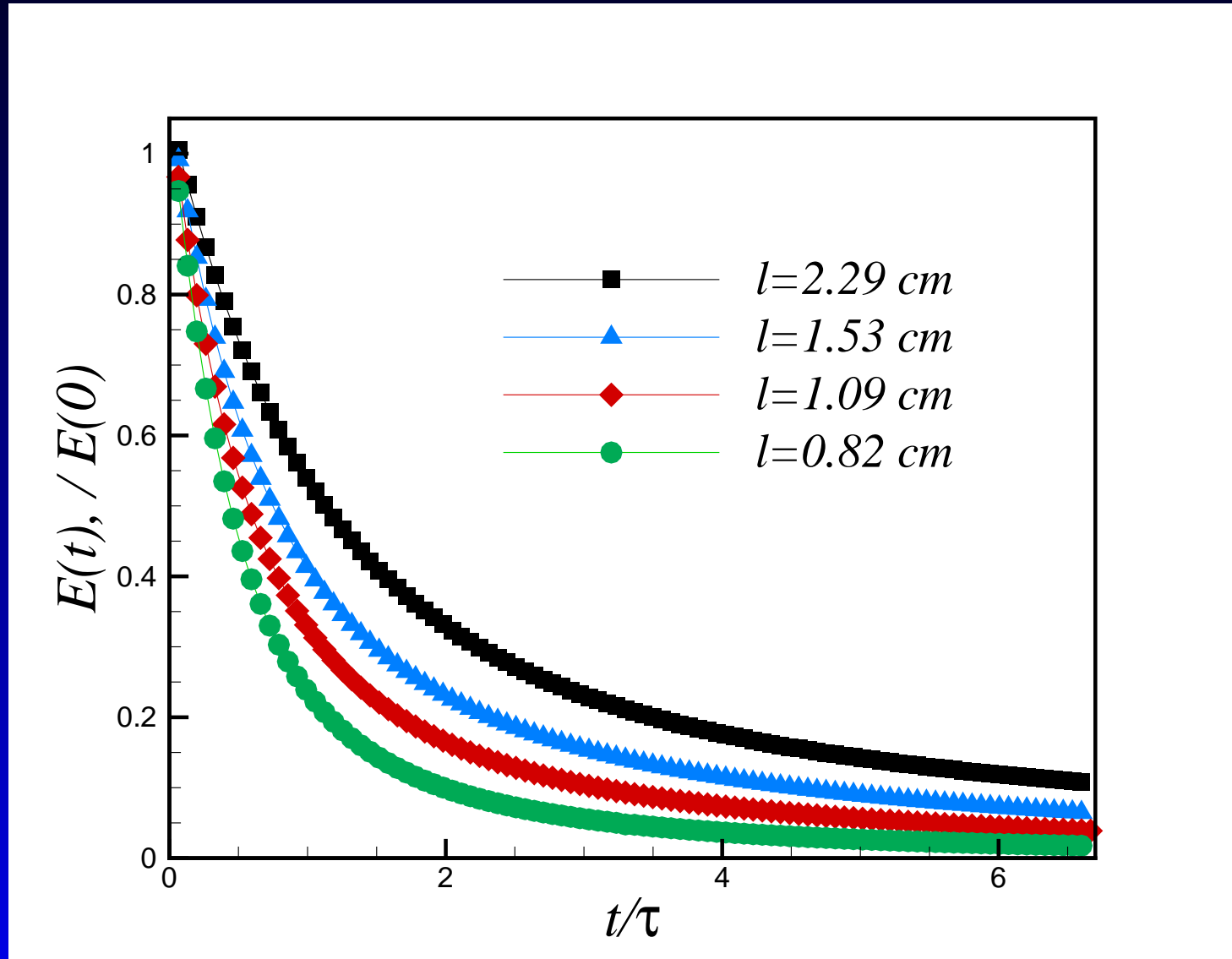
$$\ell_1/\ell_2 = 2.8$$

$$t/\tau = 0$$

Movie: $E(t)$



Turbulent kinetic energy decay



Large and small scale decay

Turbulent kinetic energy is divided into a large-scale and a small-scale content:

$$E_S(t) = \int_0^{k_s} E(k, t) dk$$

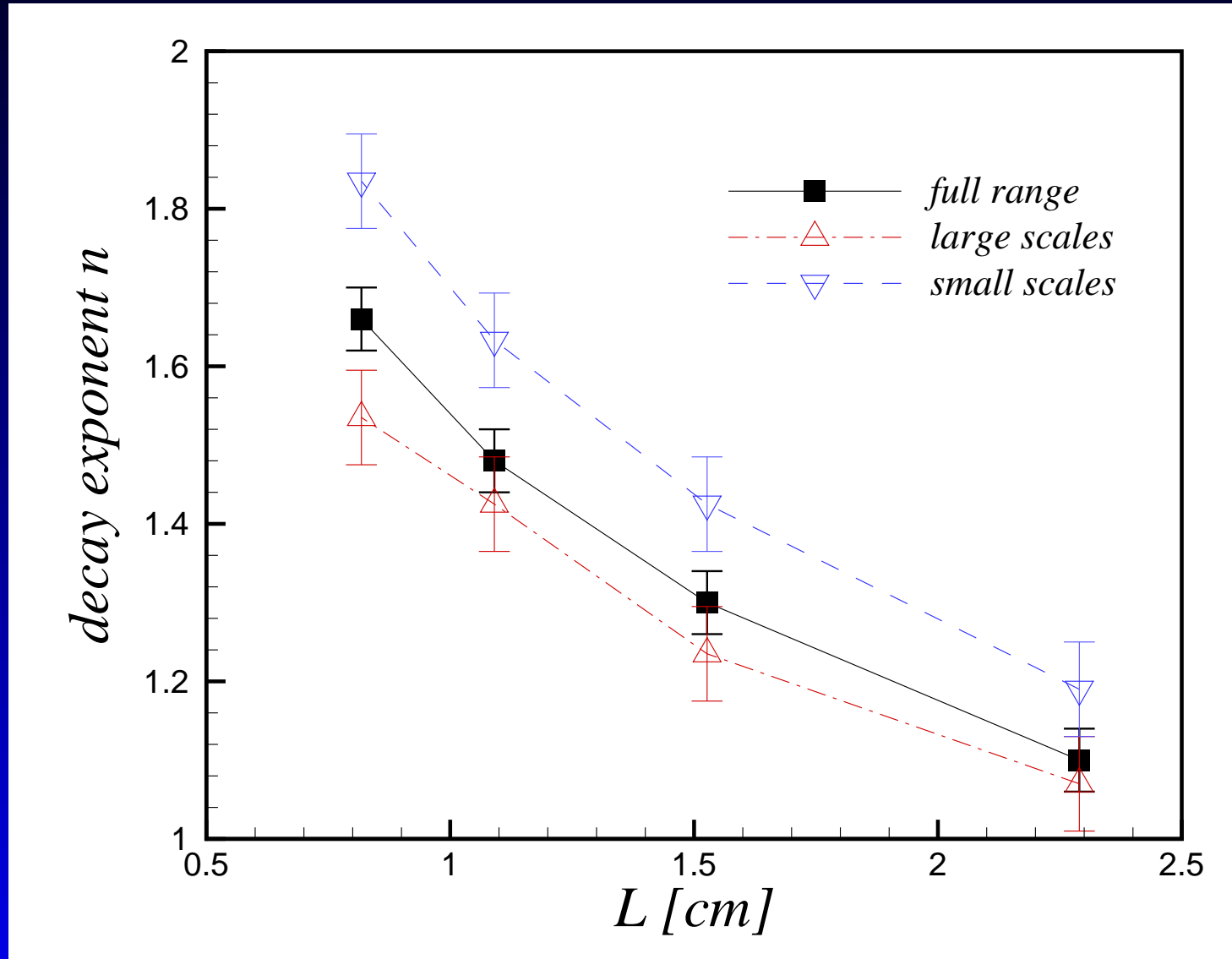
and

$$E_L(t) = \int_{k_s}^{+\infty} E(k, t) dk$$

k_s is chosen so that

$$\begin{aligned} E_L(0) &= 0,6E \\ E_S(0) &= 0.4E \end{aligned}$$

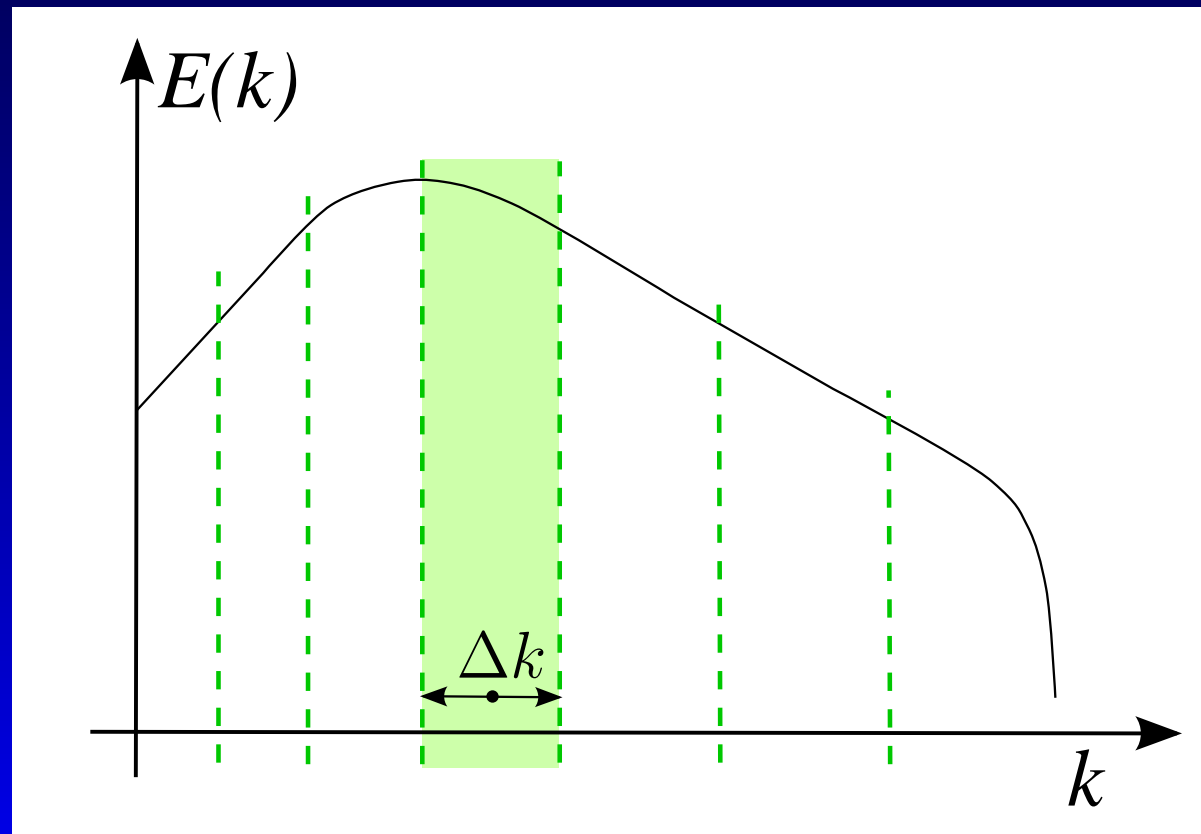

Decay exponent



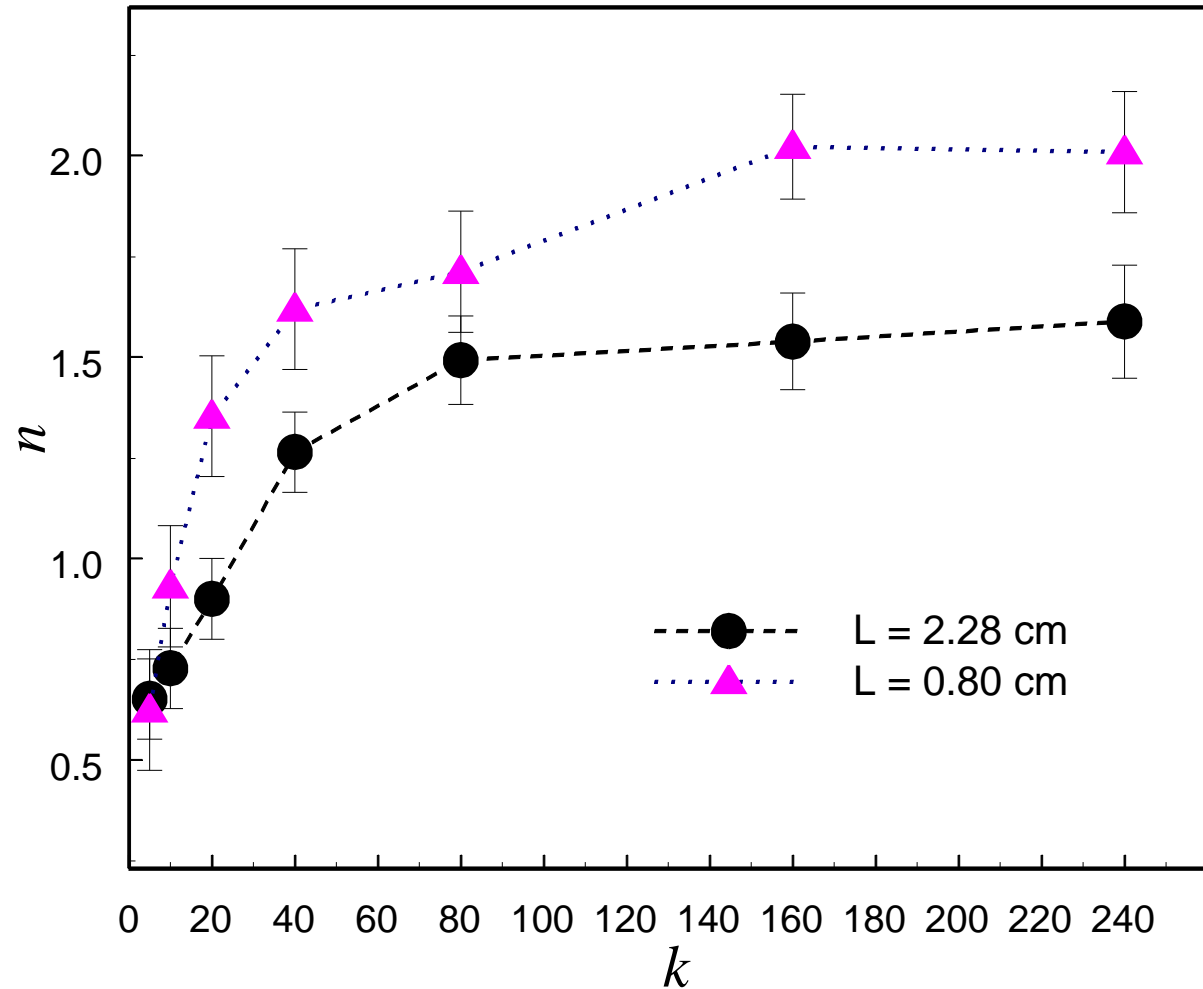
Scale by scale exponent

We measure the decay exponent scale by scale:

$$E(k, t) \approx t^{-n(k)}$$

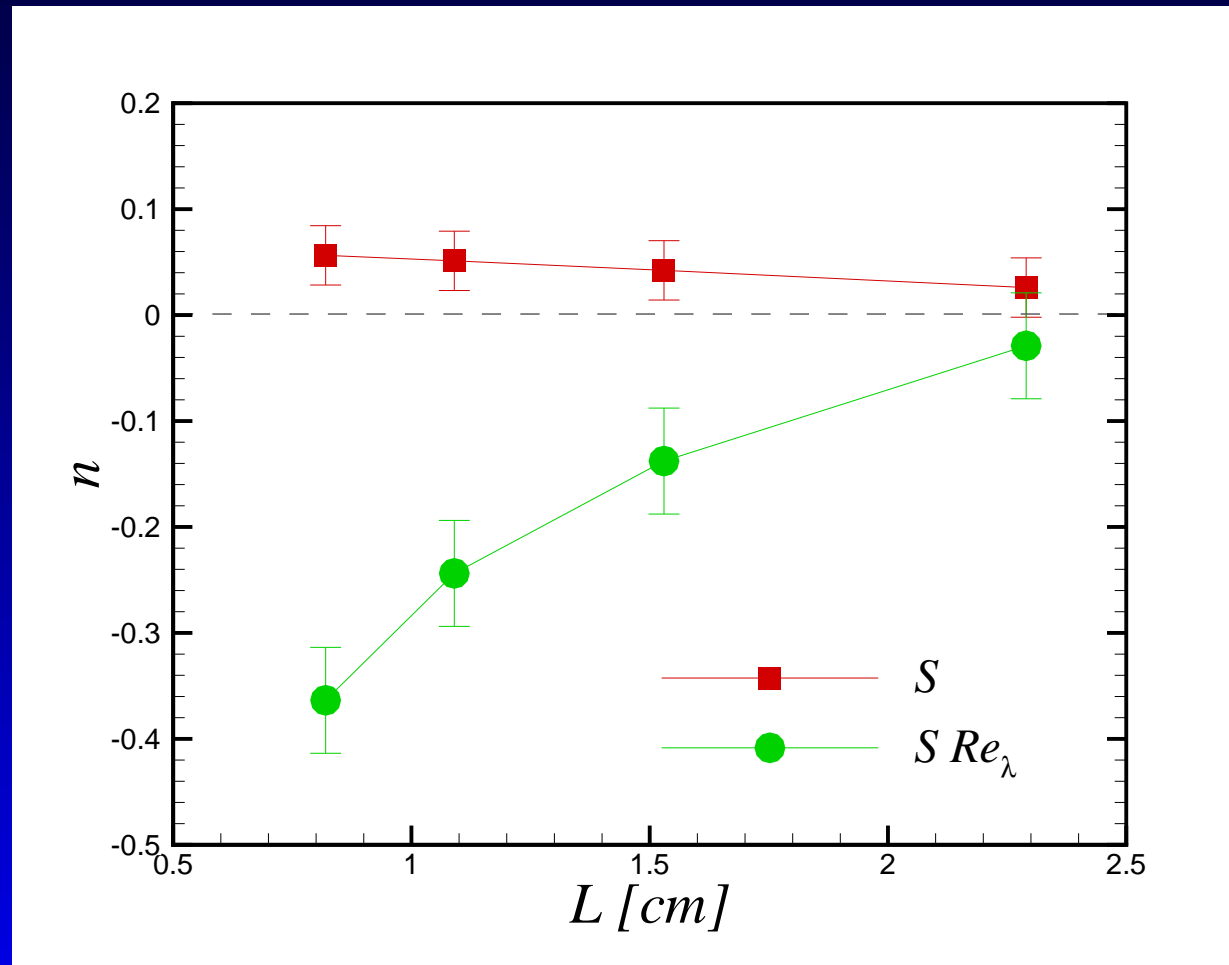


Scale by scale exponent



Derivative skewness

$$S_{\partial u/\partial x} \sim t^a, \quad S_{\partial u/\partial x} Re_\lambda \sim t^b$$



Conclusions

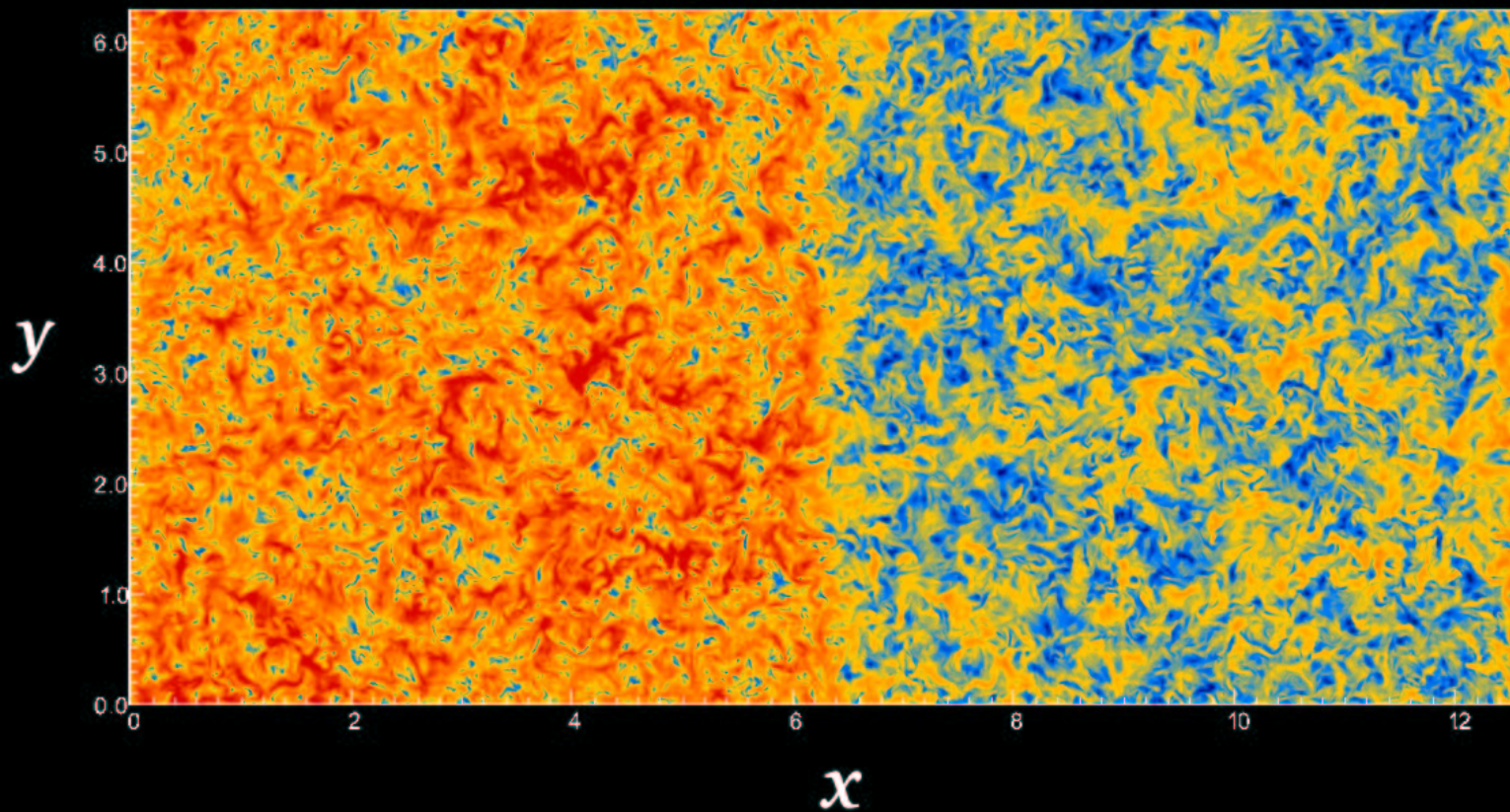
Simple numerical experiments on decaying homogeneous and isotropic turbulence show:

- decay exponent is affected by large and small scale organizations
- decay exponent is closer to -1 are associated with turbulence where more of the energy is distributed at low wavenumbers
- derivative skewness remains constant



Uniform integral scale

Kinetic energy ratio = 6.7, uniform integral scale



Movie: $E(t)$



Diffusion of a passive scalar across a turbulent energy gradient

M.Iovieno, L.Ducasse, D.Tordella

Politecnico di Torino, Dipartimento di Ingegneria Aeronautica e Spaziale

EFMC 8, Bad Reichenhall, September 2010



Passive scalar

Basic phenomenology

- A passive scalar is a contaminant present in so low concentration that it has no dynamical effect on the fluid motion,
- Turbulence transports and disperses the scalar by making particles follow chaotic trajectories, it stretches and folds lines of constant concentration, and scalar fluctuations reach the smaller scales.

Passive scalar

Basic phenomenology

- at large scales:
 - the mean concentration, variance and flux are strongly influenced by the boundary conditions and scalar injection
- at small scales:
 - scalar differences are not gaussian,
 - intermittency observed at inertial range scales as well as at the dissipation scales, with ramp/cliff structures

see, e.g.:

Warhaft *ARFM* 2000,

Shraiman and Siggia, *Nature* 2000,

Gotoh, *PoF* 2006, 2007.

Turbulent shearless mixing

Diffusion of a
passive scalar
across a
turbulent
energy
gradient

Introduction

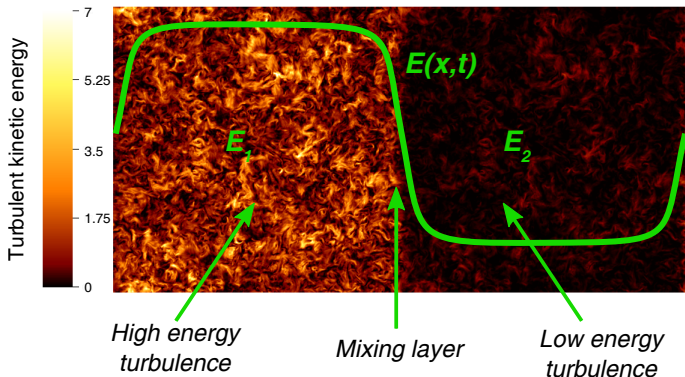
Passive scalar

Mean Scalar

Scalar
moments

Conclusions

General flow configuration:



periodic boundary condition \Rightarrow 2 mixing layers



Main features

Shearless mixing layers shows the following properties:

- no gradient of mean velocity, thus no kinetic energy production
- the mixing is generated by the inhomogeneity in the turbulent kinetic energy and integral scale
- the mixing layer becomes very intermittent at both large and small scales (EC-512, 2009)
- the presence of a gradient of energy is a sufficient condition for the onset of intermittency (Phys.Rev.E, 2008, Phys.Rev.Lett, subm.2010)
- 2D and 3D mixings: different asymptotic layer thickness growth exponent



Passive scalar transport

We solve the passive scalar advection-diffusion equation

$$\frac{\partial \vartheta}{\partial t} + u_j \frac{\partial \vartheta}{\partial x_j} = \frac{(-1)^{n+1}}{Re Sc} \nabla^{2n} \vartheta$$

for the shearless mixing configuration.

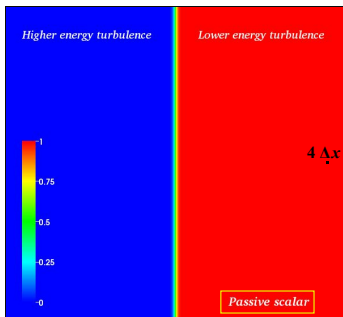
DNS simulations have been performed at $Re_\lambda = 150$ and $Sc = 1$, both in 3D turbulence ($600^2 \times 1200$ grid, $n = 1$) and 2D turbulence (1024^2 grid, $n = 2$).



Scheme of the flow

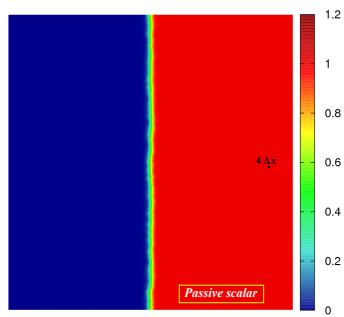
Passive scalar

3D Mixing
($600^2 \times 1200$ grid)



Run 3D Movie

2D Mixing
(1024^2 grid)



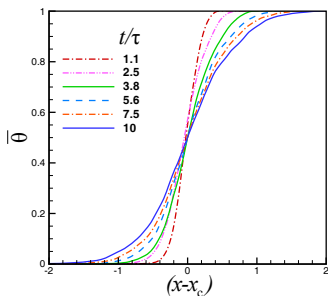
Run 2D Movie

The passive scalar is initially introduced in the low energy turbulent region and diffuses through the mixing layer

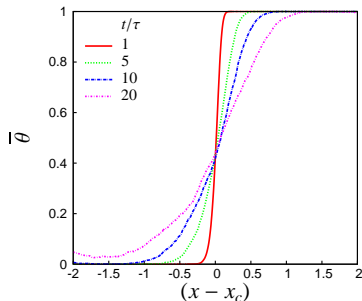


Mean Scalar Diffusion

3D Mixing



2D Mixing

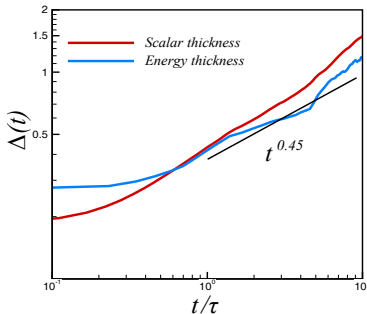


Energy ratio $E_1/E_2 = 6.7$, Schmidt number = 1.

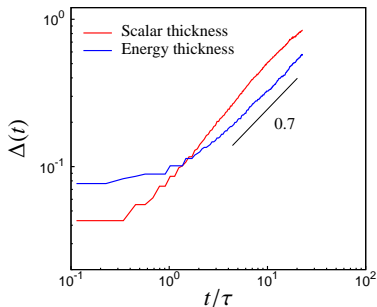


Scalar mixing layer thickness

3D Mixing



2D Mixing

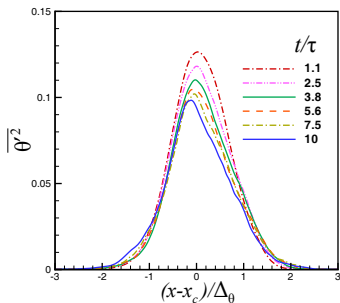


Scalar layer thickness: $\Delta_{\vartheta} = x_{\vartheta=0.75} - x_{\vartheta=0.25}$

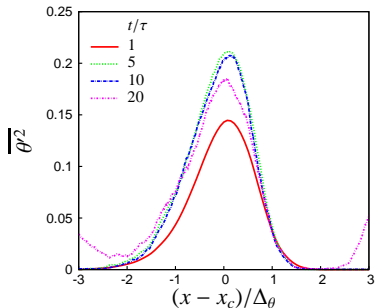
3D mixing: $\Delta_{\vartheta} \sim t^{0.45}$, 2D mixing: $\Delta_{\vartheta} \sim t^{0.7}$

Scalar variance

3D Mixing



2D Mixing

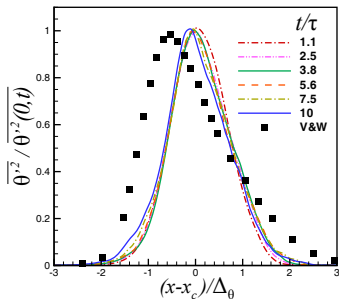


Self-similar distribution, peak shifted toward the high kinetic energy region

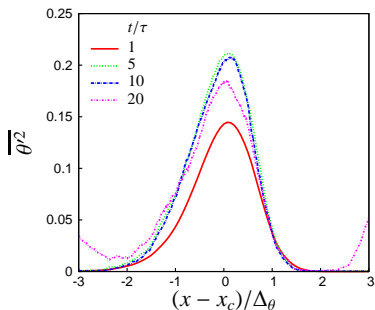


Scalar variance

3D Mixing



2D Mixing

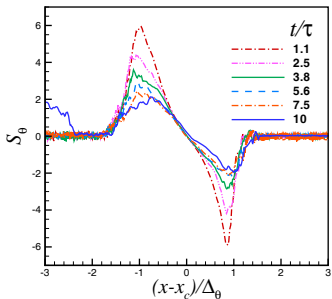


Veeravalli and Warhaft, 1990: laboratory experiment, **linear source** in the mixing layer centre, data at $x/x_0 = 0.4$ ($t/\tau \approx 4$).

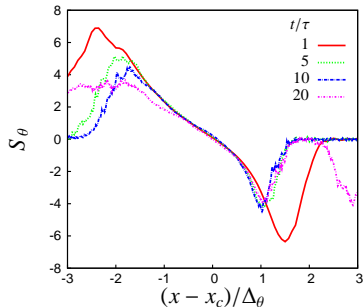


Scalar skewness

3D Mixing



2D Mixing



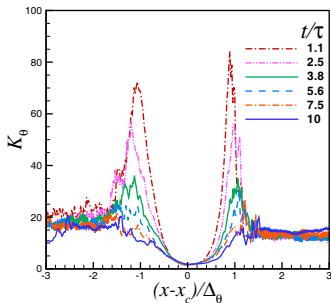
Strong non-gaussian statistic at the mixing layer border

2D: intermittency penetrates more in the direction opposite to the energy gradient.

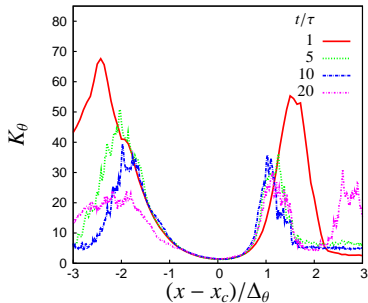


Scalar kurtosis

3D Mixing



2D Mixing



2D: higher asymmetry, wider intermittent region

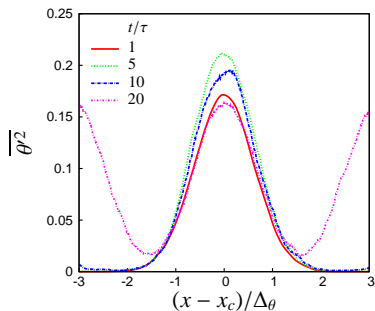
Intermittency gradually reduces as the mixing proceeds



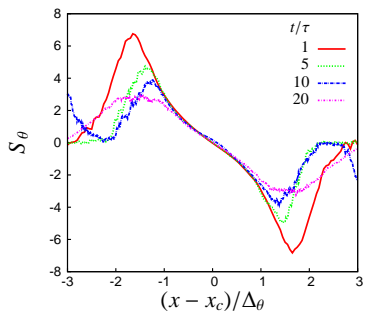
No energy gradient

2D mixing - numerical validation

Scalar variance



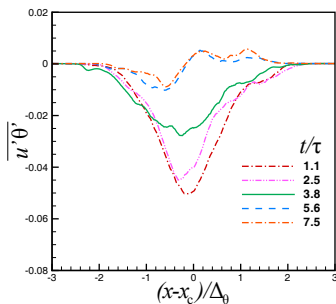
Scalar skewness



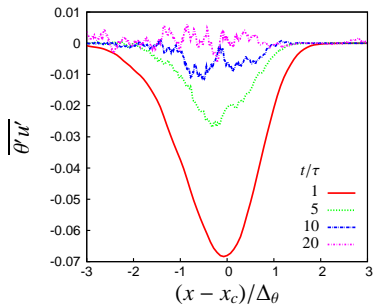
No energy gradient \Rightarrow no asymmetry

Scalar flux

3D Mixing



2D Mixing



$$\overline{u'\vartheta'} \sim 1/\Delta_\vartheta(t)$$



Conclusions

2D/3D Passive scalar diffusion across an energy step:

- all moments profiles are skewed towards the higher kinetic energy region
- self-similar profiles of first and second order moments
- large intermittency and non-gaussian behaviour on both sides of the mixing, even where the scalar flux is small.
- larger asymmetry in moment distributions in 2D mixing
- 2D: no stretching, inverse cascade, long-range interaction which penetrate more against the energy gradient



A measure of turbulent diffusion in two and three dimensions



F. De Santi¹, **L. Ducasse**¹,
J. von Hardenberg²,
M. Iovieno¹, D. Tordella¹

¹Politecnico di Torino, Torino, Italy

²Istituto di Scienze dell' Atmosfera e del Clima, CNR, Torino, Italy

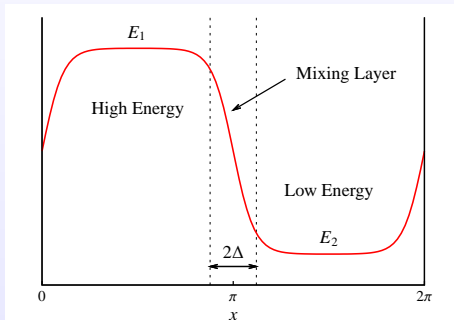
September 14, 2010

European Fluid Mechanics Conference - 8

Presentation of the problem

2 turbulent flows put aside with different kinetic energies :

- ▶ a **high** energy field on the **left** of energy E_1
- ▶ a **low** energy field on the **right** of energy E_2



Mixing layer thickness : $\Delta(t)$

$\Delta(0) \approx l$ (integral scale)

$l \approx D/80$

Periodic boundary conditions : 2 mixing layers in the simulation

Presentation of the problem

Main goals :

- ▶ Study the turbulent diffusion through the evolution in time of the mixing layer
- ▶ Compare 2D and 3D cases

Presentation of the problem

Main goals :

- ▶ Study the turbulent diffusion through the evolution in time of the mixing layer
- ▶ Compare 2D and 3D cases

Shearless mixing layers show the following properties:

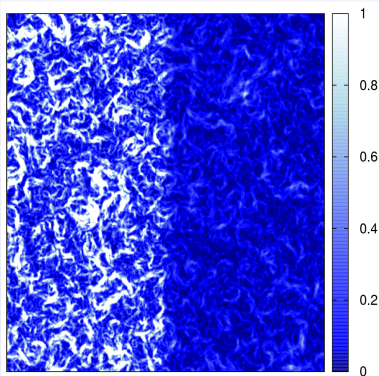
- ▶ No gradient of mean velocity → no kinetic energy production
- ▶ Mixing generated by the inhomogeneity in the turbulent kinetic energy
- ▶ Intermittent behavior at both large and small scales (EC-512, 2009)
- ▶ Gradient of energy : sufficient condition for the onset of intermittency (Phys.Rev.E, 2008)
- ▶ 2D and 3D mixings → show a very different behaviour

A visualisation

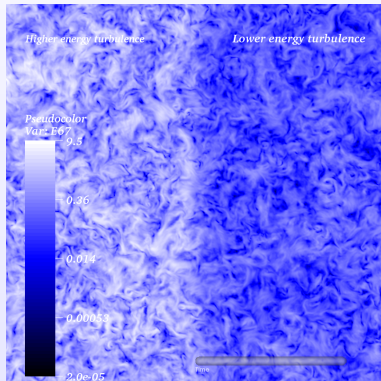
Kinetic energy : evolution in time

Initial energy ratio : $E_1/E_2 = 6.6$

2 D



3 D



Important remarks

Main parameter : Initial energy ratio E_1/E_2

The system has been studied using the values :

$$E_1/E_2 = 6.6, 40, 300, 10^4, 10^6$$

In the Navier Stokes equation :

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{\rho} \nabla p + (-1)^{p+1} \nu_n \Delta^{2n} \mathbf{u}$$

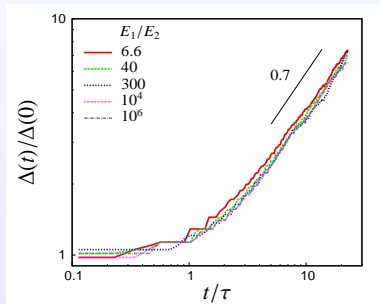
2D : An hyperviscous coefficient ($n = 2$) has been used

3D : The total energy decays faster than in **2D**

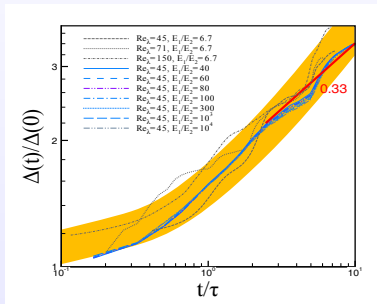
Evolution of the mixing layer

Time evolution of the mixing layer thickness $\Delta(t)$:

2 D



3 D

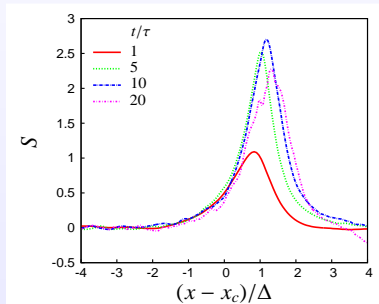


⇒ 2D mixes faster !

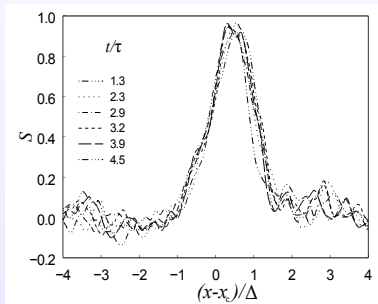
Velocity statistics

Skewness (computed along the homogeneous y direction)

2 D



3 D

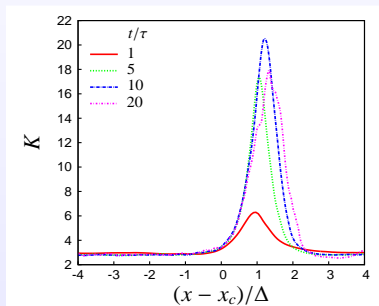


$$E_1/E_2 = 10^4$$

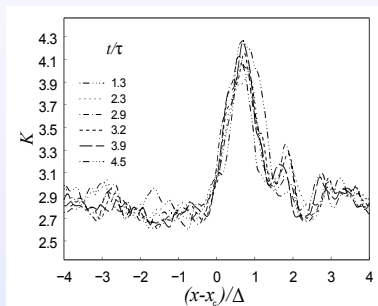
Velocity statistics

Kurtosis (computed along the homogeneous y direction)

2 D



3 D

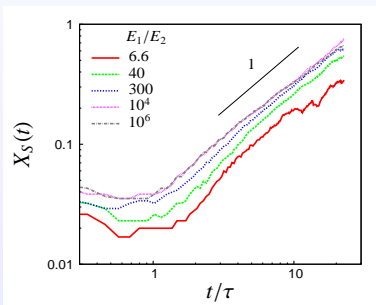


$$E_1/E_2 = 10^4$$

Velocity statistics

Position of the maximum of skewness X_S

2 D

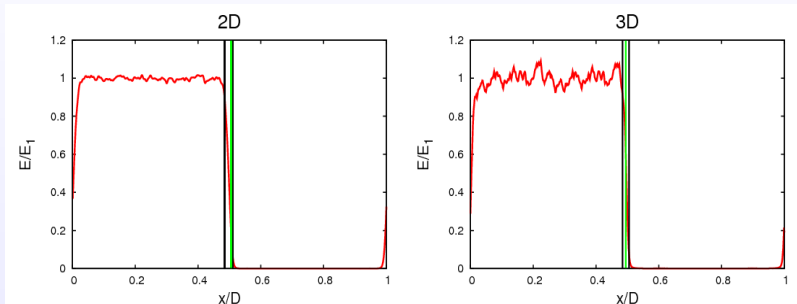


2D $\Rightarrow X_S(t) \propto t$ evolves faster than $\Delta(t) \propto t^{0.7}$

3D $\Rightarrow X_S(t) \propto \Delta(t) \propto t^{0.33}$

Time evolution

Time evolution of the energy profile :



— Mixing layer

— Position of the maximum of skewness

Total time in both cases : $\sim 22 \tau$

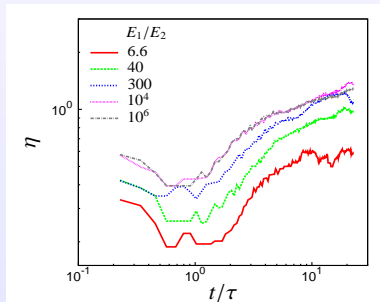
Velocity statistics

Evolution of the penetration $\eta = X_S/\Delta$

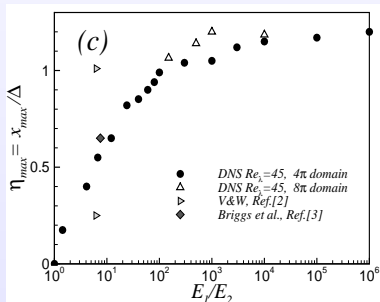
2D $\Rightarrow \eta(t)$ diverges

3D $\Rightarrow \eta(t)$ reaches a constant value : η_{max}

2 D



3 D



Memory

Proposal of a memory measure **as a global quantity referred to its own time derivative**, for example

$$MEM = \frac{\Delta}{\Delta'}$$

$$2D : \frac{d\Delta(t)}{dt} \sim t^{-0.3}, \quad 3D : \frac{d\Delta(t)}{dt} \sim t^{-0.67}$$

$$2D : \mathbf{MEM} = \frac{\Delta(t)}{\Delta(t)_t} \sim \mathbf{1.4t}, \quad 3D : \mathbf{MEM} = \frac{\Delta(t)}{\Delta(t)_t} \sim \mathbf{3t}$$

different dimensionality, same trend (qualitative universality?), with a different coefficient

3D has a slightly longer memory than 2D

Conclusions

Comparison between the 2D and 3D situation :

Similarities :

- ▶ $\Delta(t)$ evolves asymptotically in time as a power law
- ▶ A strong intermittency \rightarrow visible on the high order moments

Differences :

- ▶ Mixing is faster in 2D
- ▶ No autosimilarity in time in the 2D case

Conclusions

Comparison between the 2D and 3D situation :

Similarities :

- ▶ $\Delta(t)$ evolves asymptotically in time as a power law
- ▶ A strong intermittency \rightarrow visible on the high order moments

Differences :

- ▶ Mixing is faster in 2D
- ▶ No autosimilarity in time in the 2D case

Possible explanation :

The evolution of $\Delta(t)$ is essentially led by the large scales

2D \rightarrow energy tends to concentrate to the large scales (inverse cascade)