

Velocity derivative statistics in shearless turbulence

Euromech colloquium 501

Daniela Tordella, Michele Iovieno

daniela.tordella@polito.it

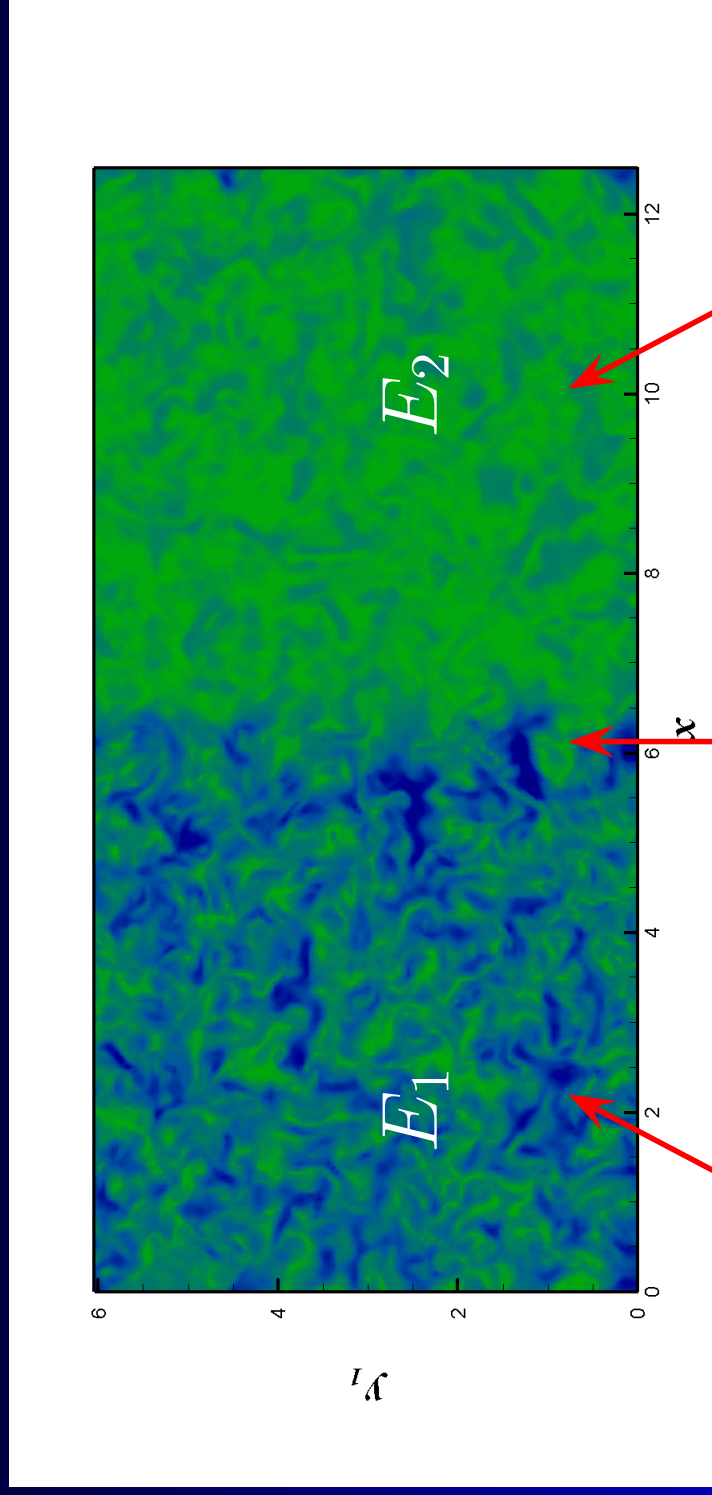
*Dipartimento di Ingegneria Aeronautica e Spaziale
Politecnico di Torino,*

Corso Duca degli Abruzzi 24, 10129 Torino, Italy



Turbulent shearless mixing

Ref: *J. Fluid Mech.* 549, 441-451, (2006).



Run Movie 1-2

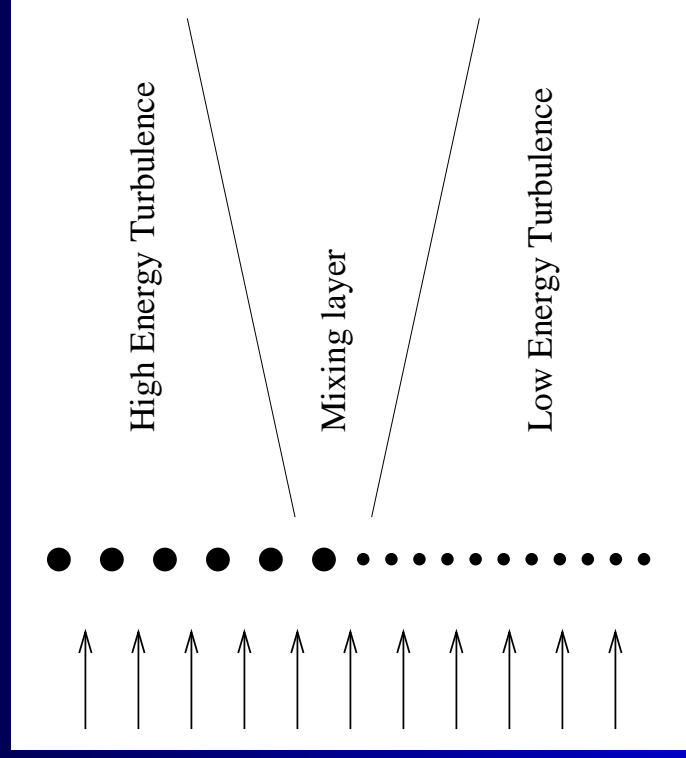
1-High energy turbulence 2-Low energy turbulence

Mixing layer



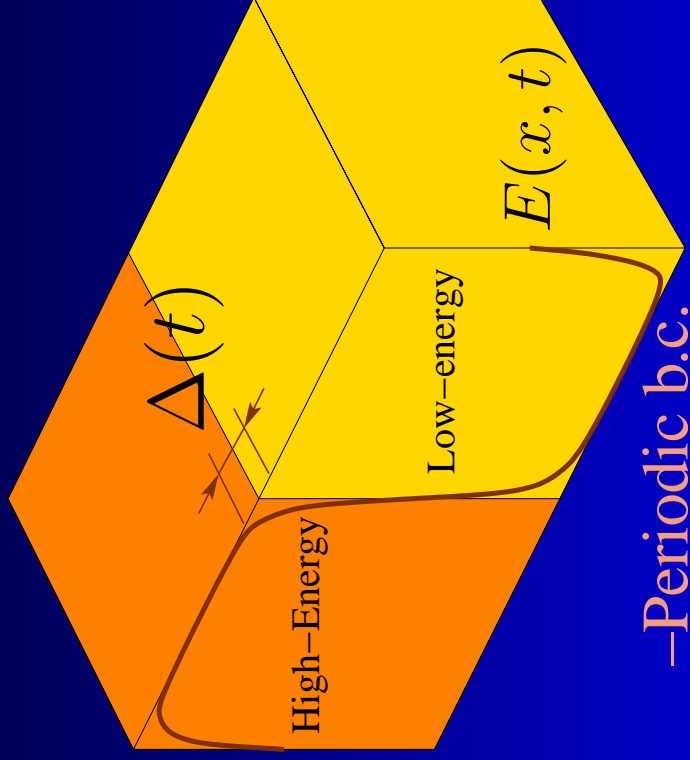
State of the art

- Grid turbulence experiments:
 - ▲ Gilbert *JFM* 1980
 - ▲ Veeravalli-Warhaft *JFM* 1989



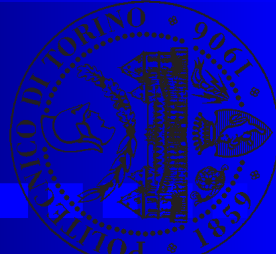
State of the art

- Grid turbulence experiments:
 - ▲ Gilbert *JFM* 1980
 - ▲ Veeravalli-Warhaft *JFM* 1989
- Numerical experiments:
 - ▲ Briggs et al. *JFM* 1996
 - ▲ Knaepen et al. *JFM* 2004
 - ▲ Tordella-Iovieno *JFM* 2006
 - ▲ Iovieno-Tordella-Bailey *PRE* 2008)



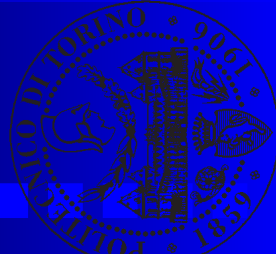
Main features

- Self-similar stage of decay
- $\overline{u_1^2} \approx \overline{u_2^2} \approx \overline{u_3^2}$
- High intermittency, function of:
 - ▶ gradient of turbulent kinetic energy
 $\Rightarrow \mathcal{E} = E_1/E_2$
 - ▶ gradient of integral scale $\Rightarrow \mathcal{L} = \ell_1/\ell_2$



Method

- We consider two kinds of shearless mixings:
 - with a *uniform scale*: the mixing is generated by the gradient of energy only $\Rightarrow \mathcal{E}$ only parameter ($\mathcal{L} = 1, \mathcal{E} \neq 1$)
 - with a *scale* and *energy* gradient: $\Rightarrow \mathcal{E} \neq 1$ and $\mathcal{L} \neq 1$
- The behaviour of the mixing in the limit of very high energy ratio \mathcal{E} is investigated.
- Method: DNS, parallelepiped domain ($2\pi \times 2\pi \times 2n\pi, n = 2$ and $n = 4$), $Re_\lambda = 45$ and 71, LES, same domain and $Re_\lambda = 44 - 450$



Velocity statistics

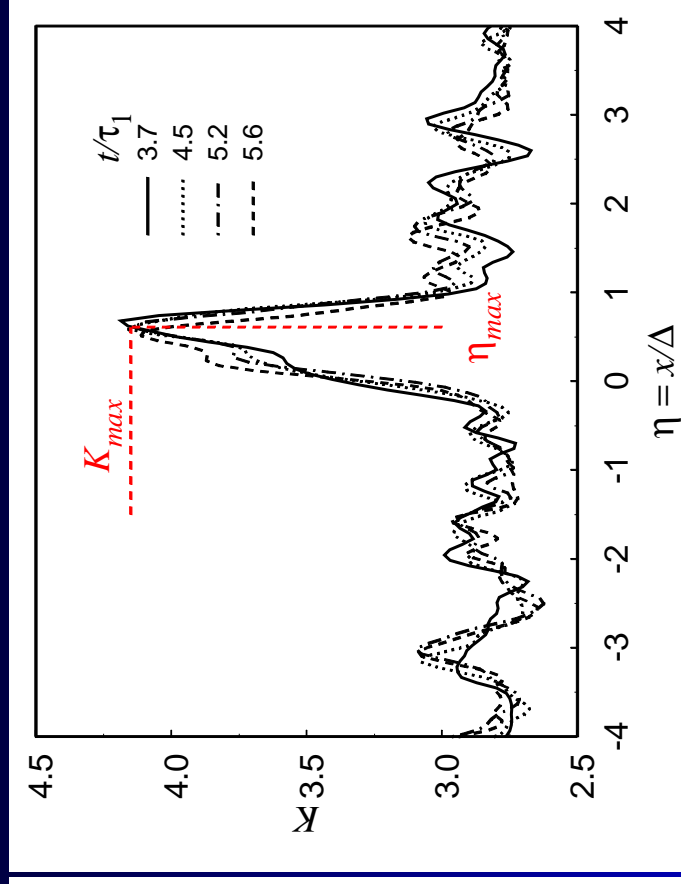
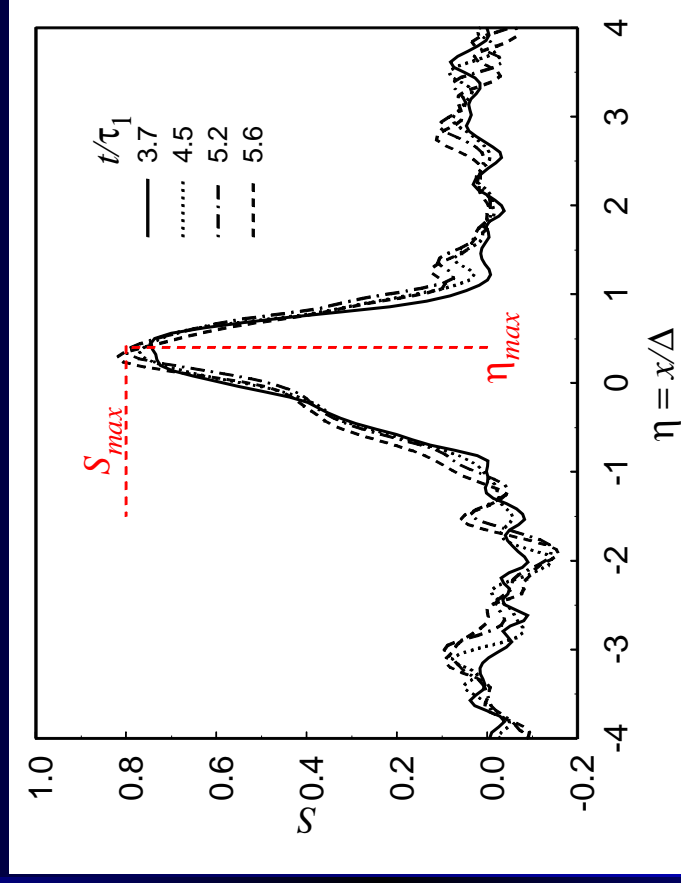
- No shear \Rightarrow no kinetic energy production,
- The mixing is intermittent even if no gradient of scale is present (*PRE* 2007, to appear)
- Intermittency is (*JFM* 2006)
 - ▶ *ENHANCED* if the energy gradient is concurrent with the integral scale gradient ($\mathcal{L} > 1$)
 - ▶ *REDUCED* if the energy gradient is opposite to the integral scale gradient ($\mathcal{L} < 1$)
- Self-similar stage of evolution



Intermittency - $\mathcal{E} = 6.7$

$$S = \overline{u^3} / \overline{u^2}^{3/2}$$

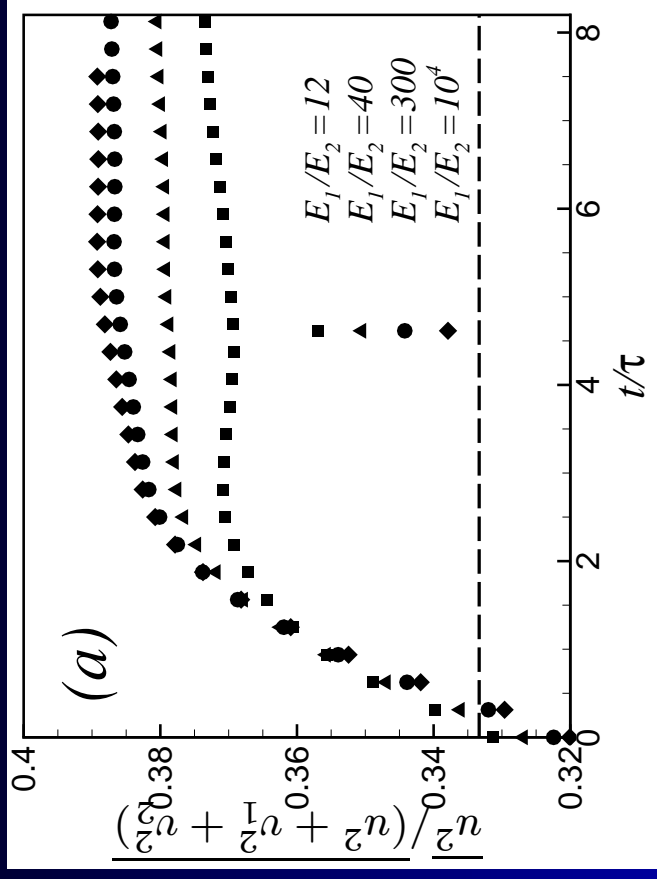
$$K = \overline{u^4} / \overline{u^2}^2$$



S_{max} , K_{max} = maximum of Skewness and Kurtosis
in the mixing layer
 η_{max} = position of the maximum in the mixing layer



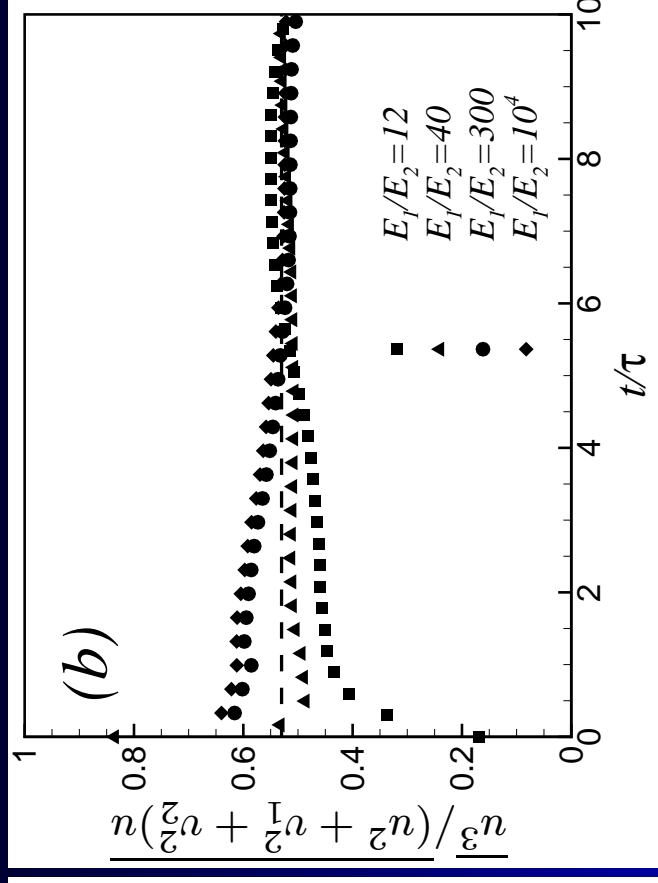
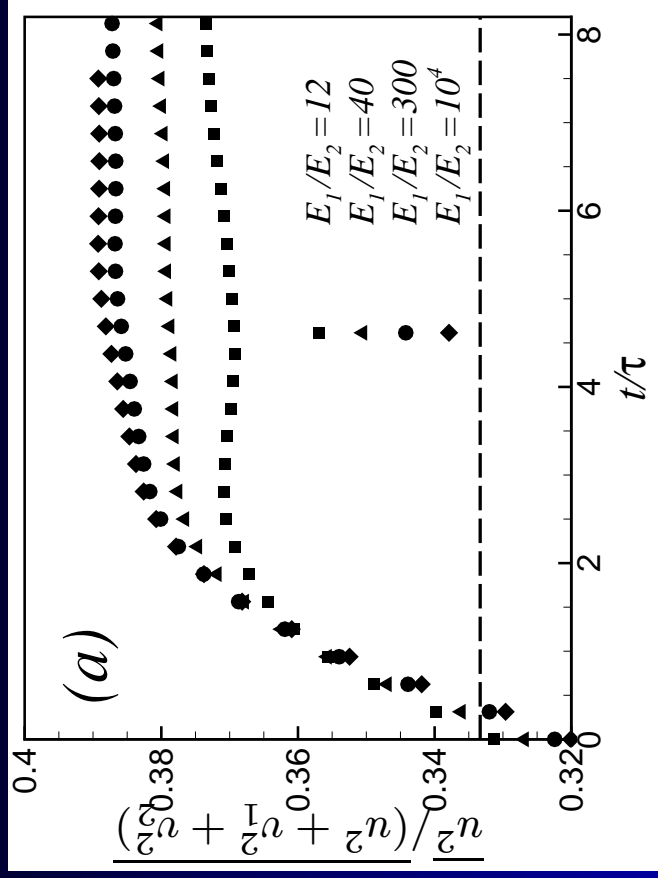
Anisotropy



Left: second order moment anisotropy



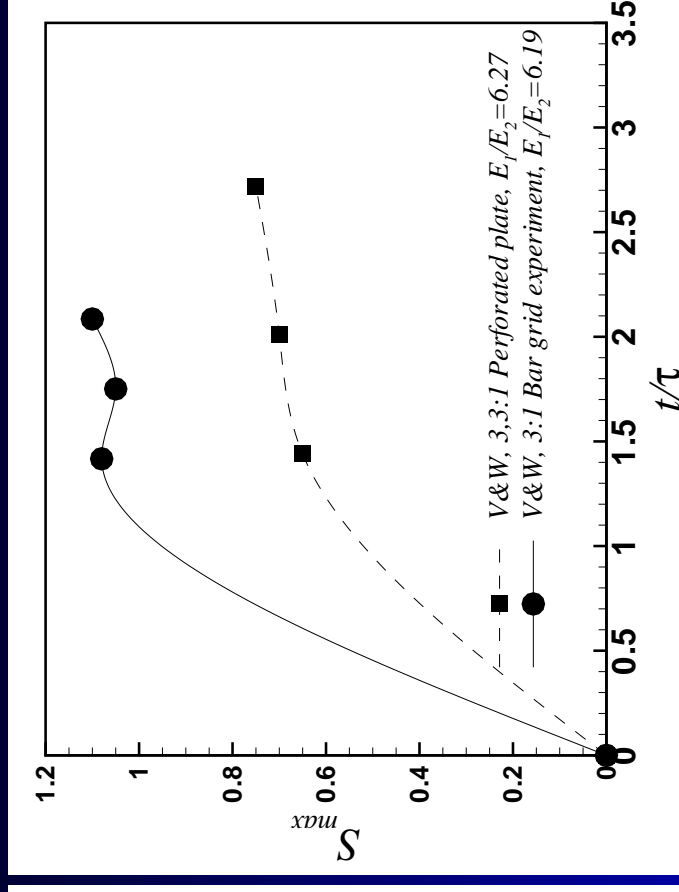
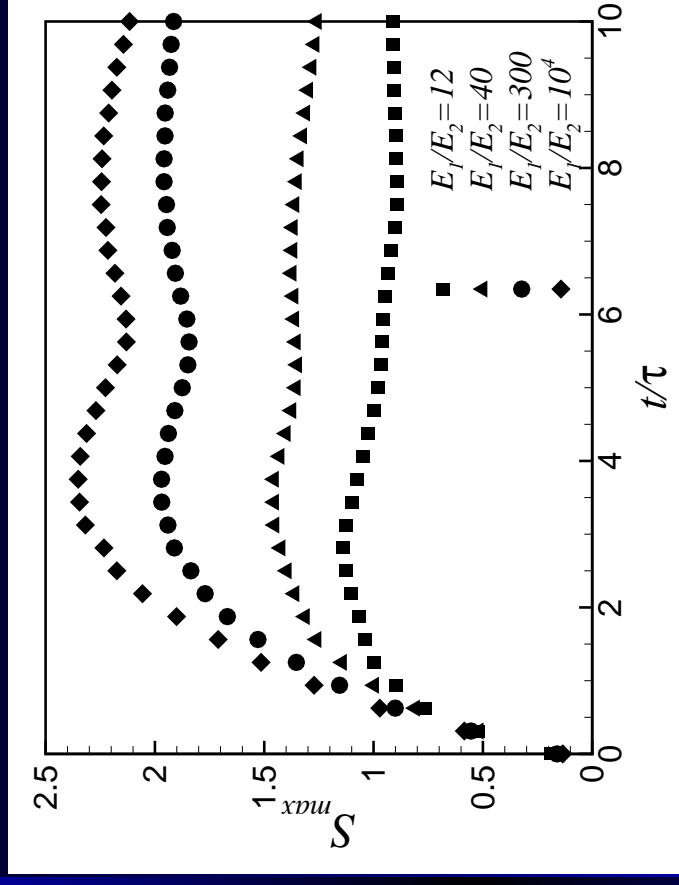
Anisotropy



Left: second order moment anisotropy
Right: third order moment anisotropy



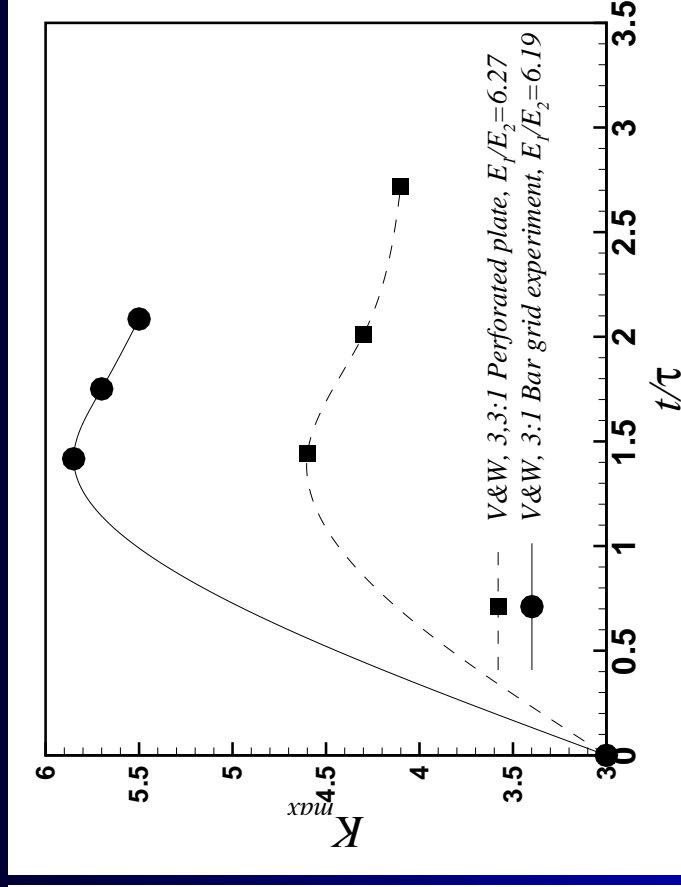
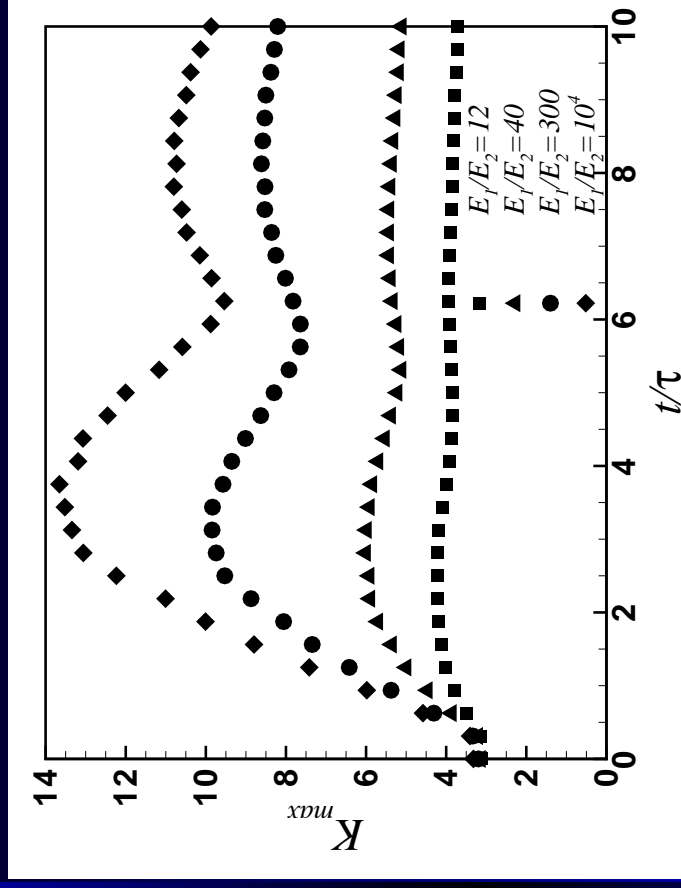
Inhomogeneity: Skewness evolution



Time evolution of the maximum of skewness within the mixing.



Inhomogeneity: Kurtosis evolution

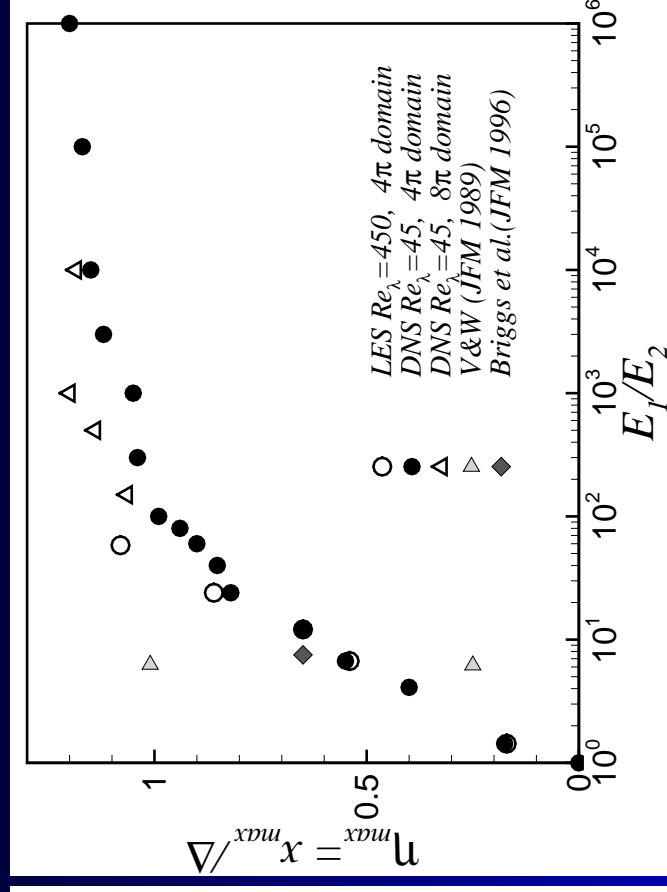
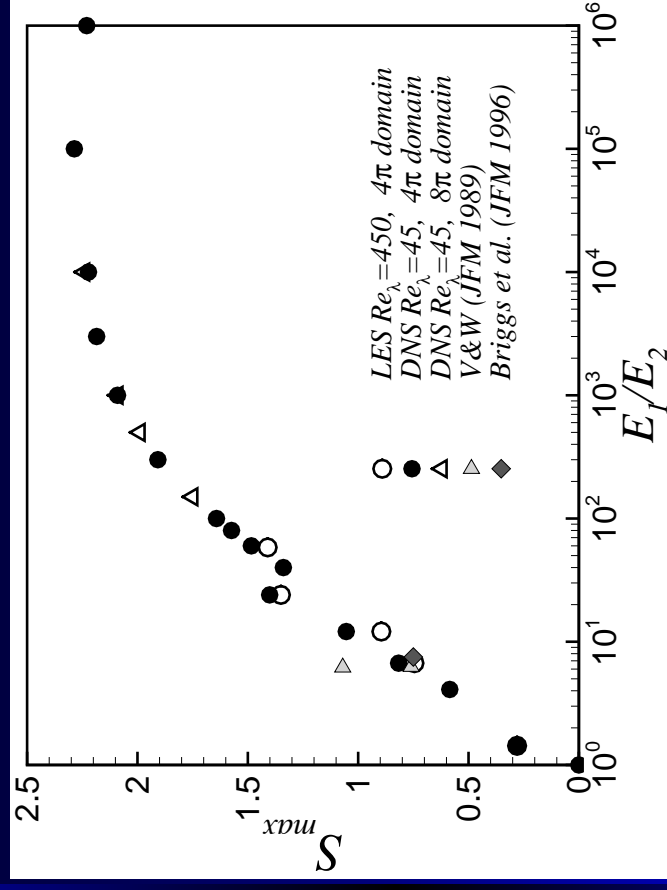


Time evolution of the maximum of kurtosis within the mixing.



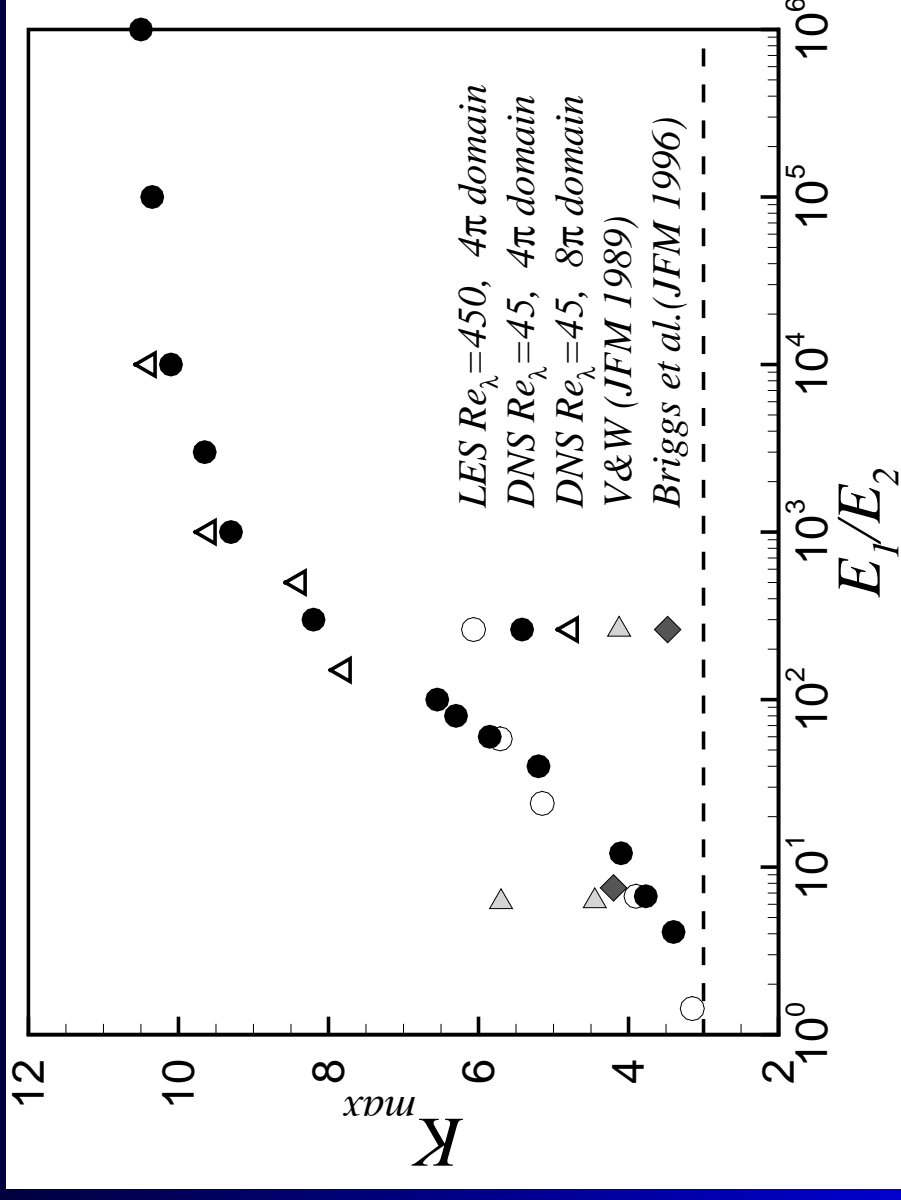
Asymptote for $E_1/E_2 \rightarrow \infty$

Skewness



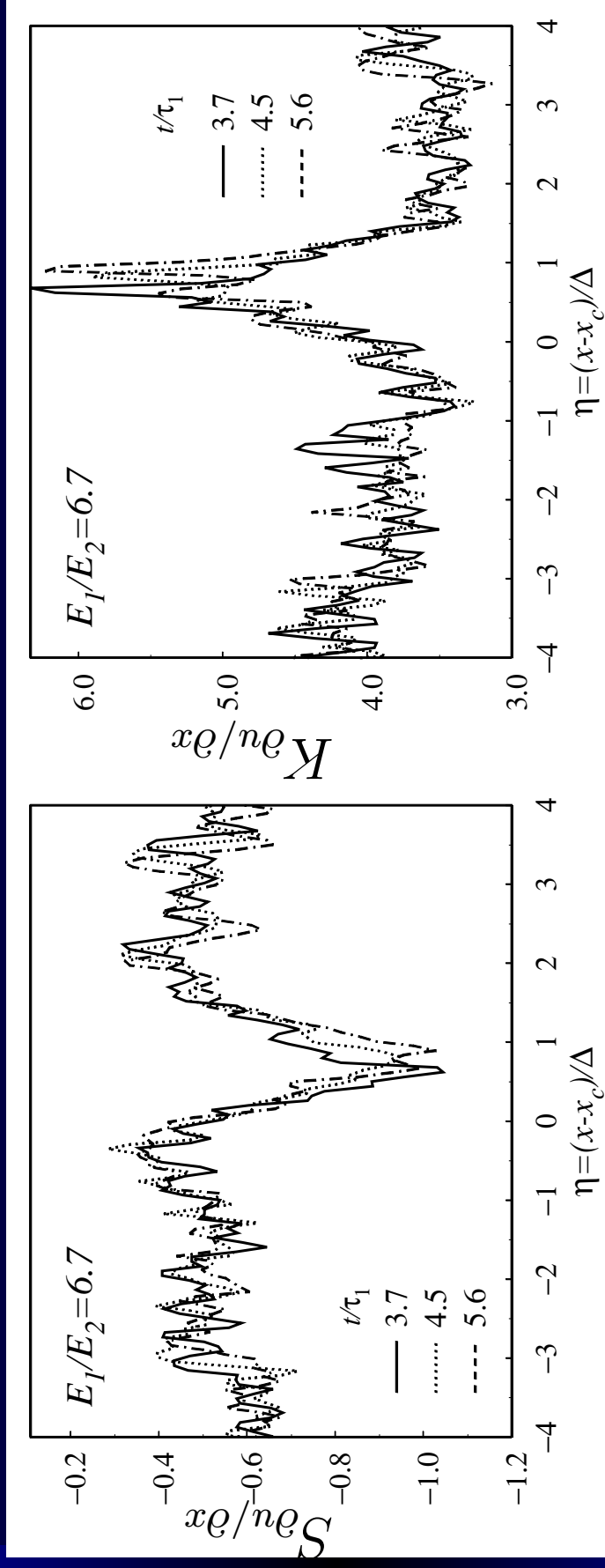
Asymptote for $E_1/E_2 \rightarrow \infty$

Kurtosis

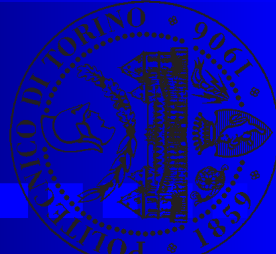


Longitudinal derivative moments distribution

$$E_1/E_2 = 6.7, \ell_1/\ell_2 = 1$$

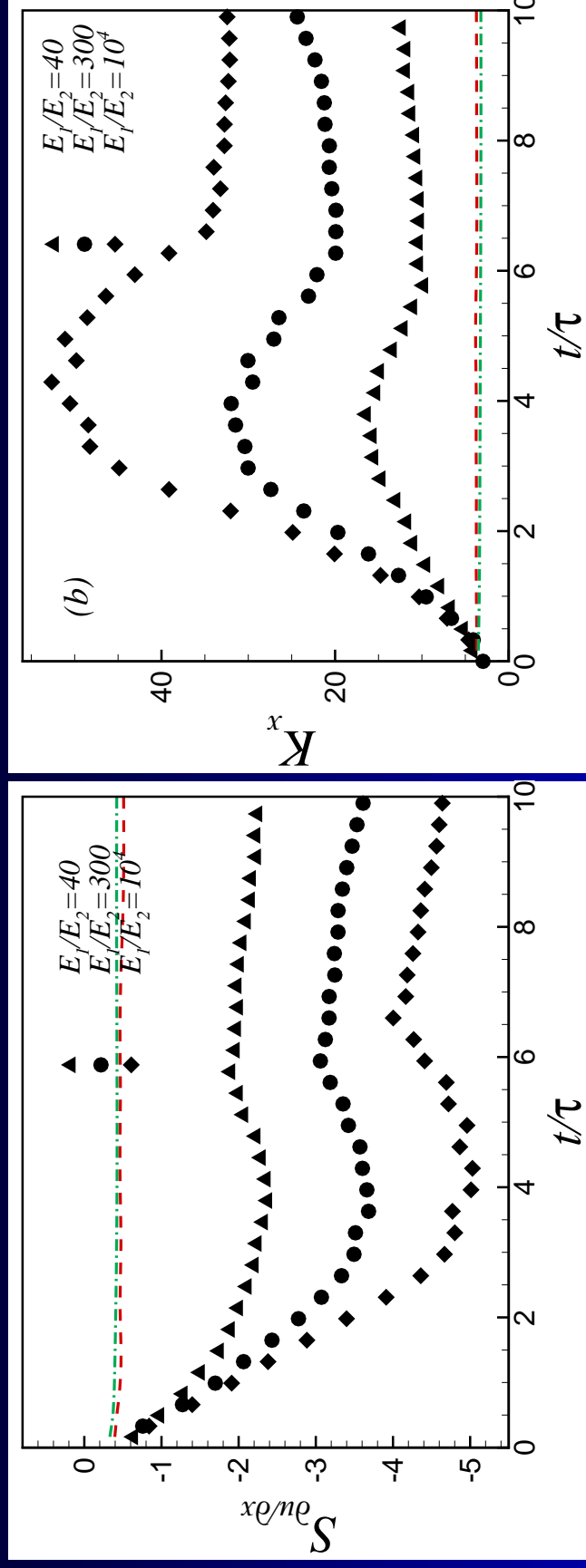


η is the dimensionless coordinate along the mixing
 Δ is the mixing half-width



Mixing: Longitudinal derivatives

Time evolution of the peak of the derivative statistics



u = velocity component in the mixing direction

x = direction along the mixing

τ = initial eddy turnover time

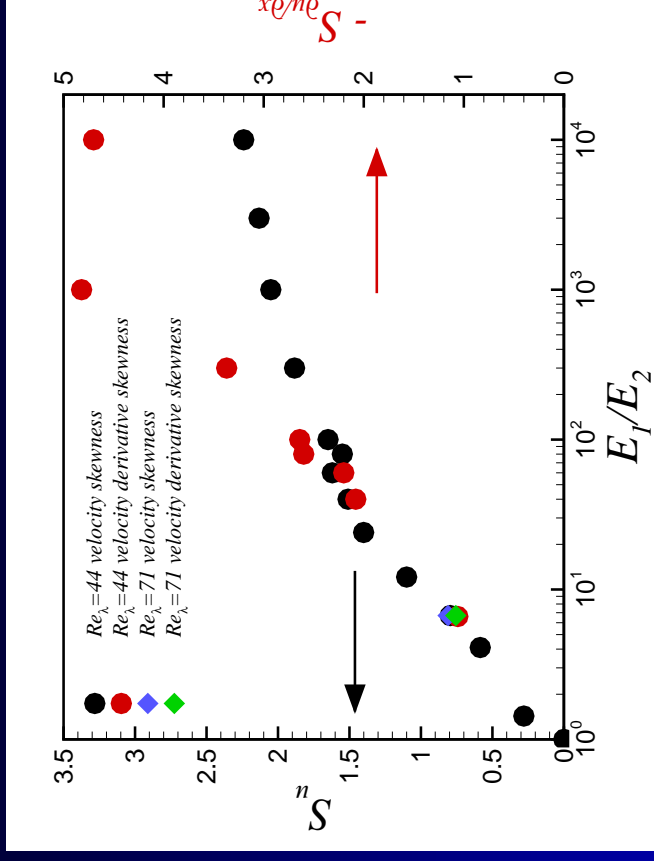
— — — high energy homogeneous region

- · - · - low energy homogeneous region

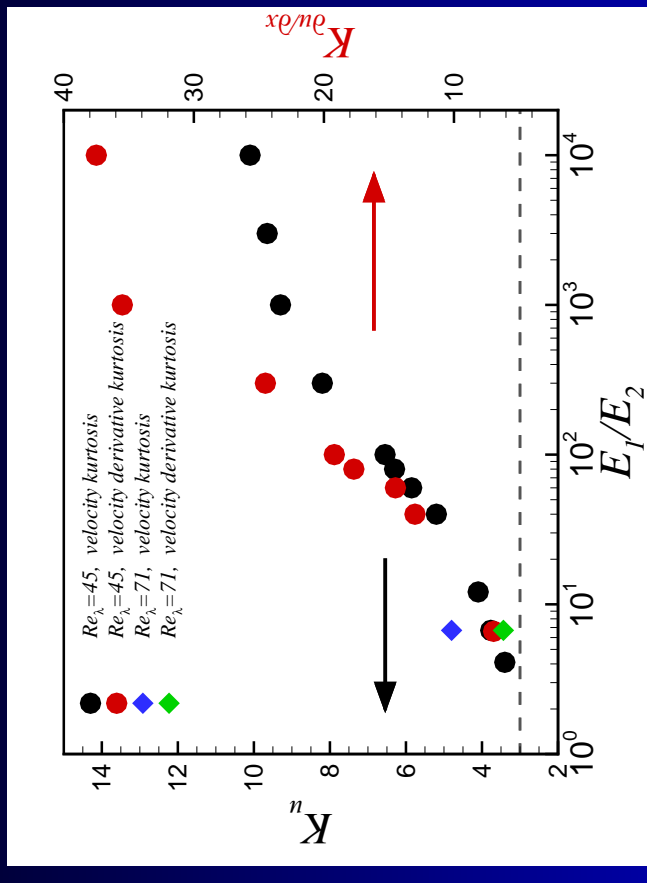


Asymptote for $E_1/E_2 \rightarrow +\infty$

Derivative Skewness



Derivative Kurtosis

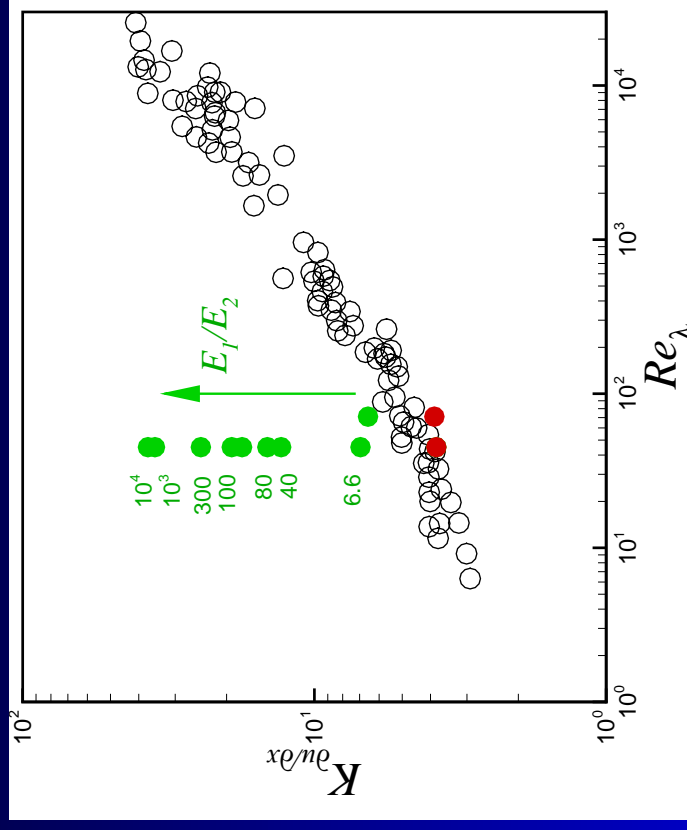
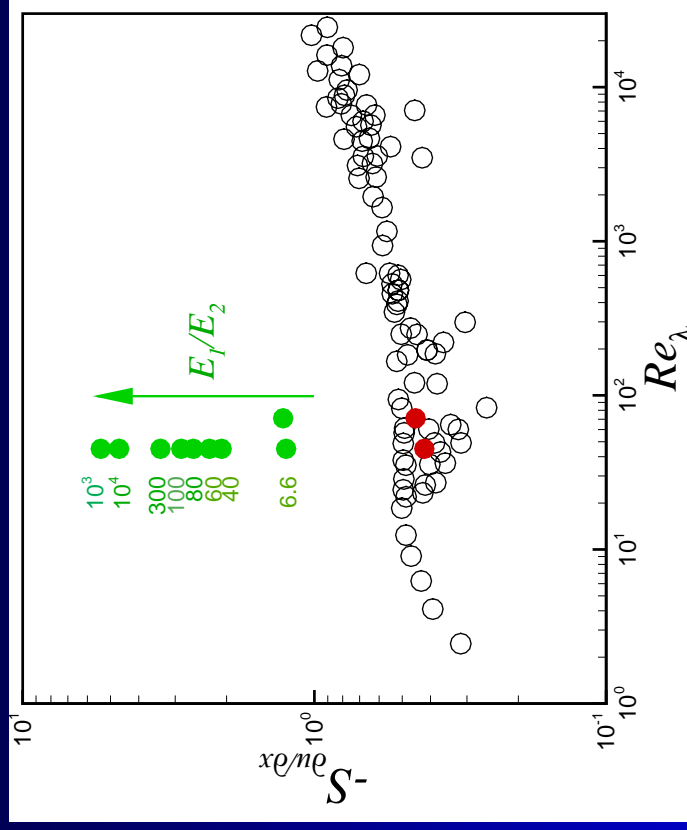


Longitudinal derivative Skewness and Kurtosis of the velocity component along the mixing direction



Comparison with homogeneous turbulence

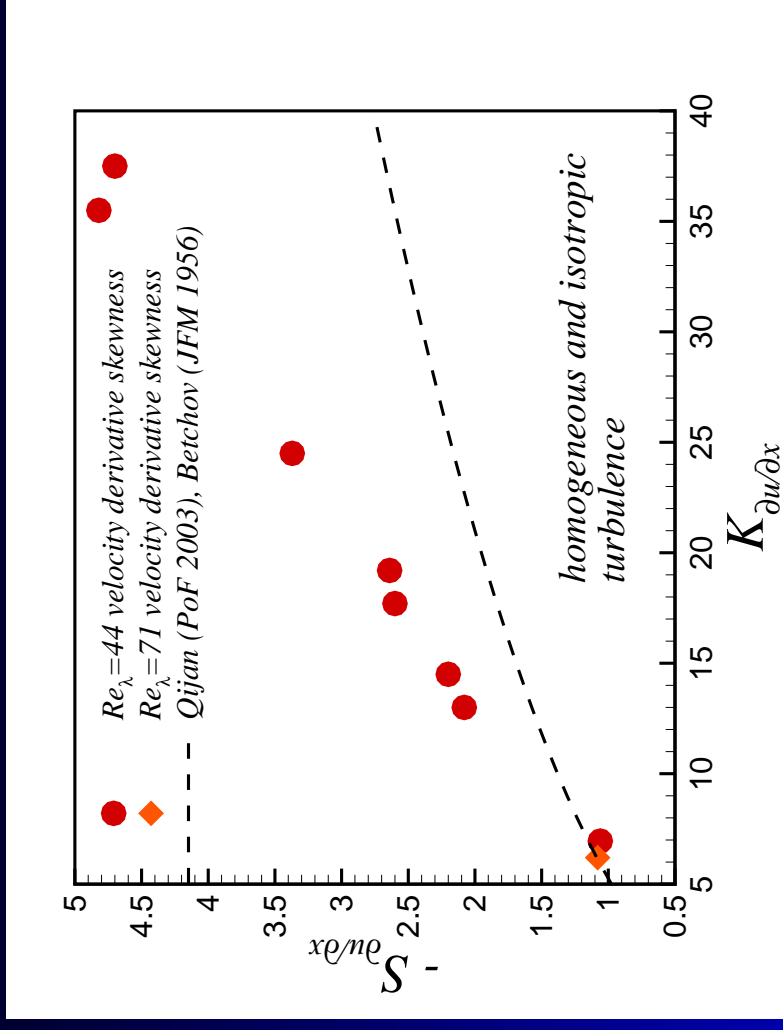
Comparison of longitudinal moments inside the mixing with longitudinal moments in homogeneous and isotropic turbulence (data from Sreenivasan and Antonia, *Ann.Rev.Fluid Mech* 1997)



- *HIT, present simulations*
- *Shearless mixings, present simulations*
- *Homogeneous turbulence*



Comparison with homogeneous turbulence (II)



Comparison with the upper bound

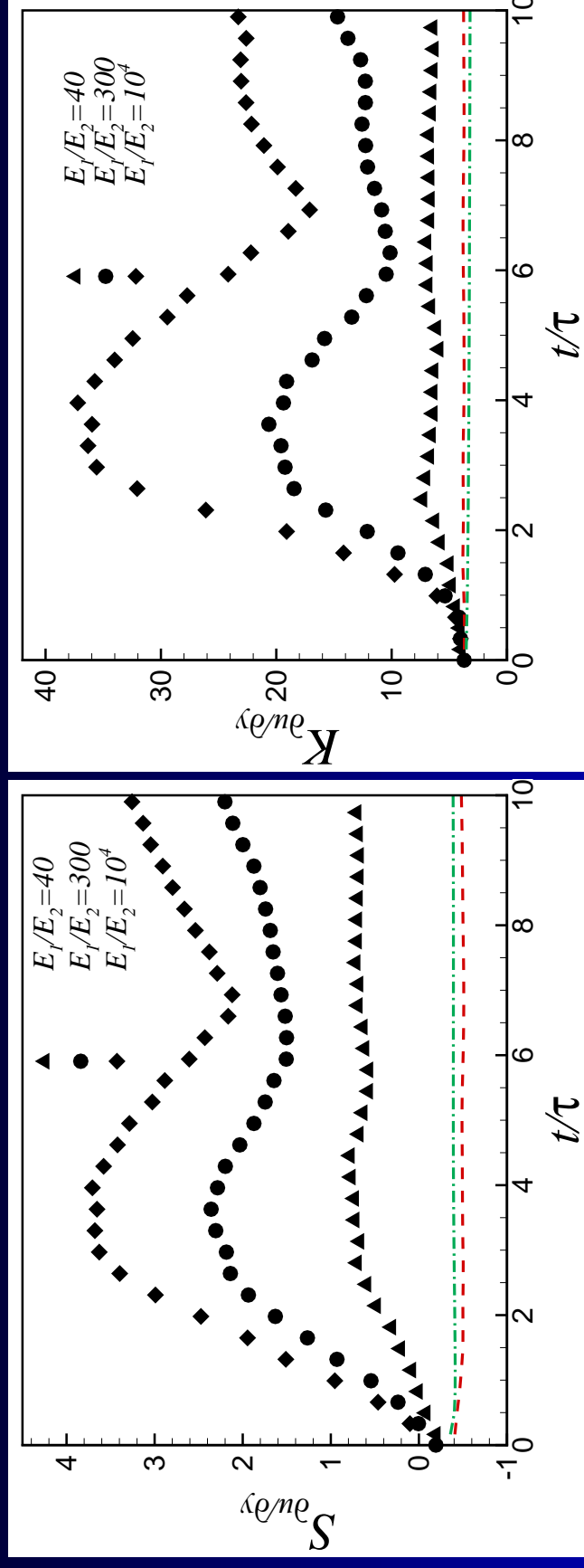
$$-S_{\partial u / \partial x} \leq 2 \left(\frac{K_{\partial u / \partial x}}{21} \right)^{\frac{1}{2}}$$

of the longitudinal skewness in homogeneous turbulence



Mixing: Transversal derivatives

Time evolution of the peak of the derivative statistics



u = velocity component in the mixing direction

y = direction normal to the mixing

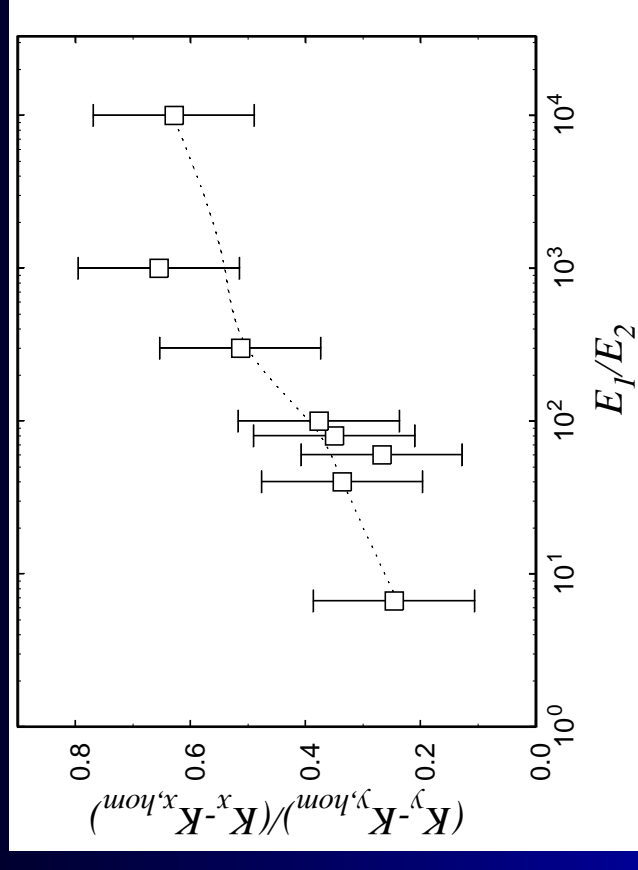
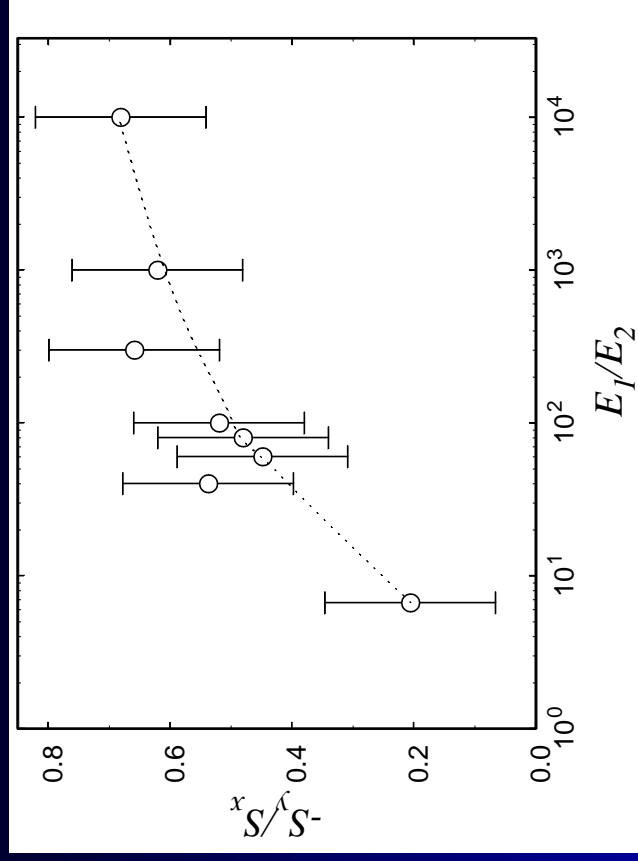
τ = initial eddy turnover time

— — — high energy homogeneous region

- · - · - low energy homogeneous region



Small scale anisotropy: uniform integral scale



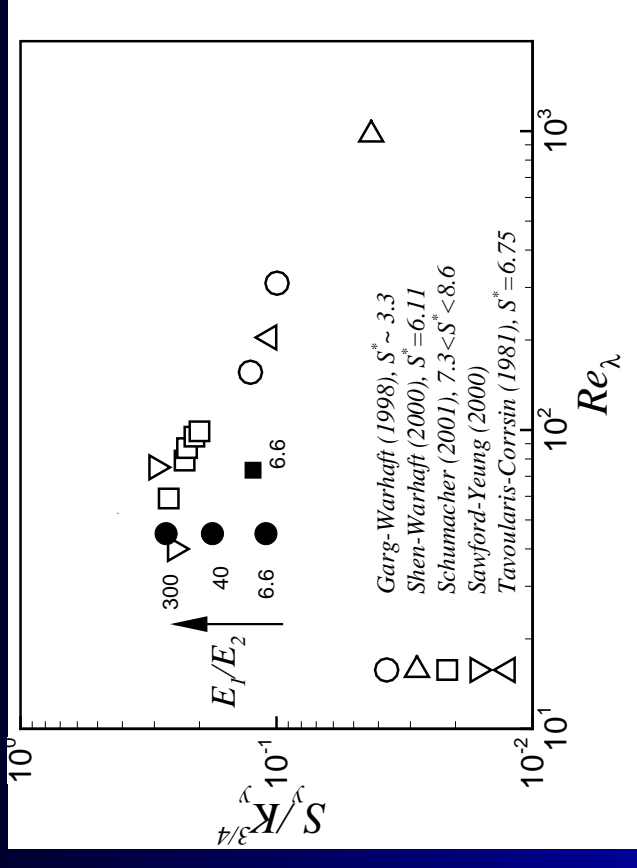
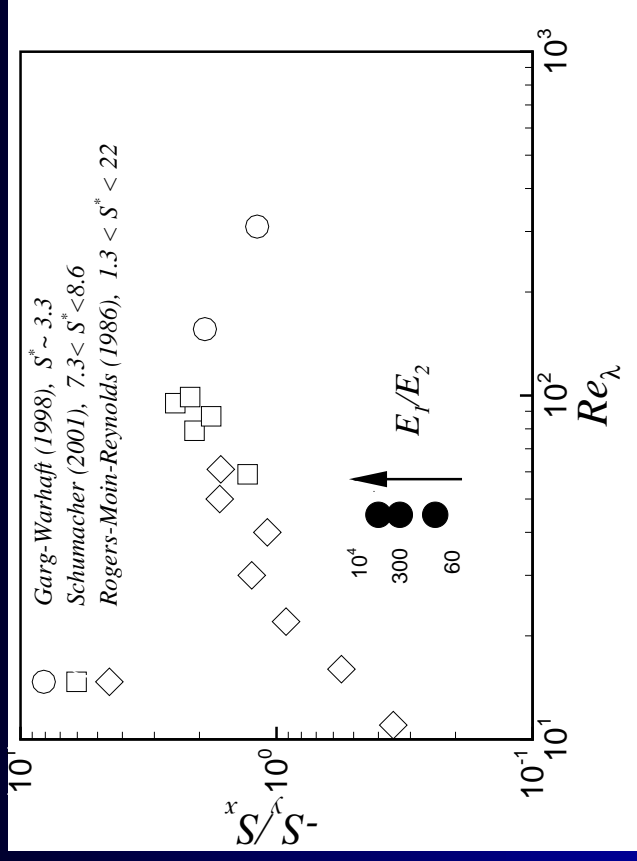
(a) isotropy of the derivative skewness;

(b) isotropy of the derivative kurtosis.

The bars represent the amplitude of the fluctuations of these ratios.



Comparison with homogeneous shear turbulence

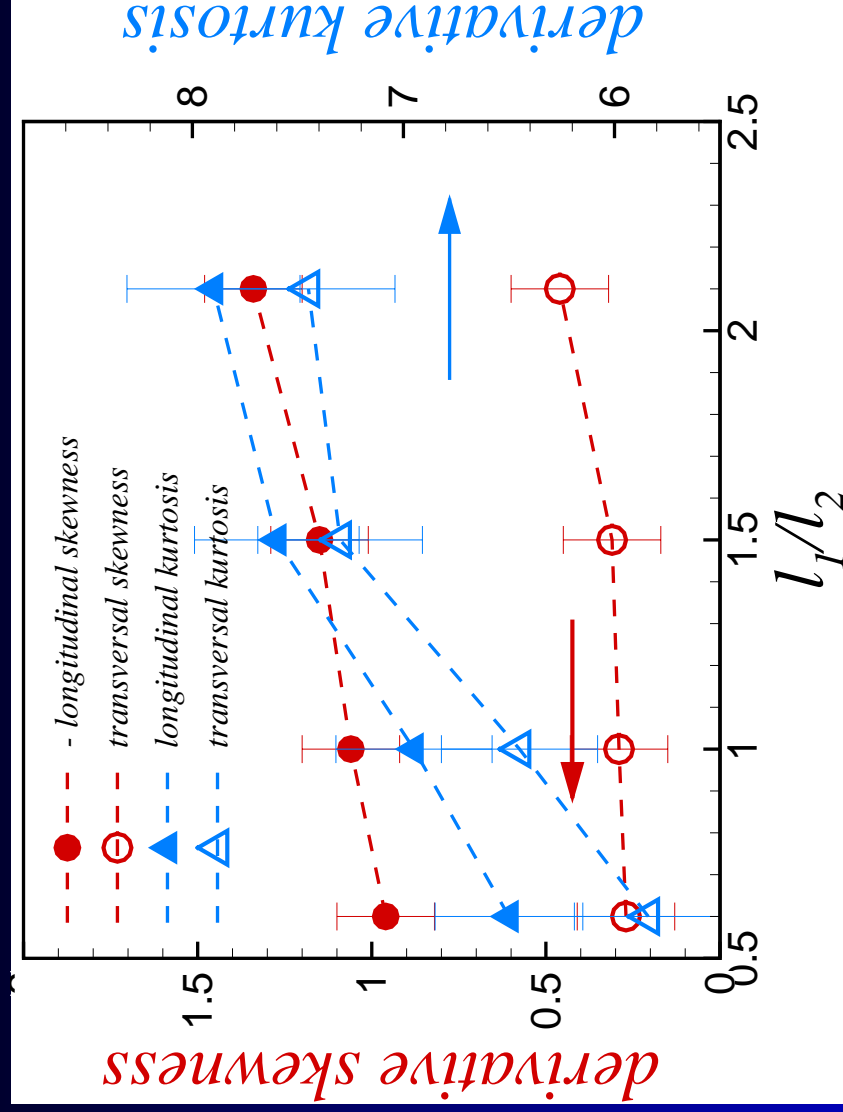


Comparison of anisotropy and intermittency:

- shearless mixing data, $\bullet Re_\lambda = 45$, $\square Re_\lambda = 71$
- empty symbols: homogeneous shear flows data.

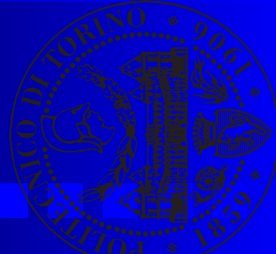


Derivative statistics: non uniform integral scale



Conclusions

- Simulations which span a large range of energy ratios, $6 \leq \mathcal{E} \leq 10^4$, show:
 - ▶ longitudinal and transversal derivative moments show a high level of intermittency, comparable with that of shear flows
 - ▶ anisotropy of third and fourth order derivative moments is ~ 0.5 and does not show any significant dependence from the energy ratio E_1/E_2 .
- A gradient of integral scale has a minor influence on the derivative statistics then on the velocity statistics



Conclusions

- Simulations which span a large range of energy ratios, $6 \leq \mathcal{E} \leq 10^4$, show:
 - ▶ longitudinal and transversal derivative moments show a high level of intermittency, comparable with that of shear flows
 - ▶ anisotropy of third and fourth order derivative moments is ~ 0.5 and does not show any significant dependence from the energy ratio E_1/E_2 .
- *Future work*
 - ▶ higher Reynolds number
 - ▶ two-point derivative statistics



Appendix: Shearless mixing statistics

$E_1/E_2 :$	$\overline{u_i u_i u_j} / u_3^{3/2}$			Velocity Kurtosis		Long. derivative skewness			Long. derivative kurtosis			Trans. Moments		
	i=1	i=2	i=3	K_{u_1}	K_{u_2}	K_{u_3}	$S_{\partial_1 u_1}$	$S_{\partial_2 u_2}$	$S_{\partial_3 u_3}$	$K_{\partial_1 u_1}$	$K_{\partial_2 u_2}$	$K_{\partial_3 u_3}$	$S_{\partial_1 u_3}$	$K_{\partial_1 u_3}$
Mixings with $\ell_1/\ell_2 = 1, R_\lambda = 45$														
6.6	0.34	0.36	0.82	3.6	3.4	4.07	-0.11	-0.10	-1.06	5.0	4.85	6.95	0.29	6.55
40	0.54	0.59	1.34	5.6	6.0	5.56	0.55	0.80	-2.08	7.1	6.77	13.0	0.50	8.50
80	0.63	0.69	1.57	6.4	6.7	6.67	0.95	1.1	-2.60	8.5	8.6	17.7	0.60	13.0
300	0.78	0.87	1.91	7.5	8.2	8.93	1.5	2.0	-3.37	16	14	24.5	1.0	13.6
10e4	0.92	0.95	2.20	7.8	8.2	11.6	3.4	3.2	-4.70	20	26	37.3	1.05	23.1
Mixings with $E_1/E_2 \approx 6.6, R_\lambda = 45$														
$\ell_1/\ell_2 :$														
0.6	0.32	0.33	0.77	3.45	3.4	3.85	-0.14	-0.10	-0.94	5.0	4.8	6.50	0.27	5.90
6.6	0.34	0.36	0.82	3.6	3.4	4.07	-0.11	-0.10	-1.06	5.0	4.85	6.95	0.29	6.55
1.5	0.49	0.52	1.20	4.0	3.6	5.5	-0.12	-0.08	-1.14	5.2	4.9	7.60	0.31	7.70
2.1	0.55	0.58	1.40	4.9	4.4	7.5	-0.05	-0.02	-1.34	5.2	5.0	7.80	0.46	8.22
Mixings with $E_1/E_2 = 6.7 \ell_1/\ell_2 = 1, R_\lambda = 71$														
	0.42	0.37	0.81	3.65	3.55	4.8	-0.15	-0.19	-1.08	4.45	4.65	6.20	0.30	5.95
Veeravalli and Warhaft(1989), $E_1/E_2 \approx 7, \ell_1/\ell_2 \approx 1.5 \div 1.7$														
bars	==	==	1.06	4.36	4.23	5.53	==	==	==	==	==	==	==	=
plate	==	==	0.63	3.47	3.49	4.07	==	==	==	==	==	==	==	=

Legend:

3 = inhomogeneous direction

1,2 = homogeneous directions

S_{u_i}, K_{u_i} = skewness and kurtosis of u_i

$S_{\partial_j u_i}, K_{\partial_j u_i}$ = skewness and kurtosis of $\partial u_i / \partial x_j$



Appendix: homogeneous shear turbulence

Homogeneous shear turbulence					Shearless mixings							
Re_λ	S^*	Sd-lg	Kd-lg	Sd-tr	Kd-tr	Re_λ	E_1/E_2	ℓ_1/ℓ_2	Sd-lg	Kd-lg	Sd-tr	Kd-tr
Garg-Warhaft 1998												
156	3.39	-0.42	7.1	0.8	11.7							
310	3.31	-0.42	6.9	0.5	8.6							
Shen-Warhaft 2000												
974	6.11	=	=	0.24	*10							
Gyffason-Warhaft 2004												
452	=	-0.546	9.94	=	=							
877	=	-0.586	13.1	=	=							
Schumacher 2001												
59	7.32	-0.74	6.3	0.96	5.6	45	6.6	1.0	-1.06	6.95	0.29	6.55
79	8.18	-0.44	4.8	0.92	6.4	71	6.6	1.0	-1.08	6.25	0.30	5.95
87	8.58	-0.50	5.3	0.90	6.4	45	40	1.0	-2.08	13.0	0.50	8.50
95	8.22	-0.37	4.3	0.92	7.2	45	300	1.0	-2.60	17.7	0.60	13.0
99	8.37	-0.40	5.0	0.87	7.1	45	10^4	1.0	-3.37	24.5	1.0	13.6
Rogers-Moin-Reynolds 1986												
11	1.3	-1.2	=	0.42	=	45	6.5	0.6	-0.94	6.50	0.27	5.90
16	3.3	-0.95	=	0.53	=	45	6.7	1.5	-1.24	7.60	0.31	7.70
22	6.2	-0.61	=	0.56	=	45	6.7	2.1	-1.34	7.80	0.46	8.22
30	10	-0.48	=	0.60	=							
40	14	-0.58	=	0.63	=							
50	19	-0.42	=	0.70	=							
61	22	-0.43	=	0.71	=							
Sawford-Yeung 2000												
40	=	=	=	0.92	*5.7							
75	=	=	=	1.20	*6.5							
Tavoularis-Corrsin 1981												
204	6.75	=	=	*0.6	*10							

Legend:

$$S^* = S \langle u^2 \rangle / \epsilon$$

dimensionless

parameter

shear

Sd = derivative skewness,

Kd = derivative kurtosis,

tr = transversal,

lg = longitudinal

