Self-similarity of the turbulence mixing with a constant macroscale gradient

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Abstract

In the absence of kinetic energy production, we consider the influence of the initial conditions characterized by the presence of an energy gradient or by the concurrency of an energy and a macroscale gradient on turbulent transport. Here, we present a similarity analysis that interpret two new results on the subject recently obtained by means of numerical experiments on the shearless mixing (Tordella & Iovieno, 2004). In short the two results are: i – The absence of the macroscale gradient is not a sufficient condition for the setting of the asymptotic Gaussian state hypothesized by Veeravalli and Warhaft (1989), where, regardless of the existence of velocity variance distributions, turbulent transport is mainly diffusive and the intermittency is nearly zero up to moments of order four. In fact, it was observed that the intermittency increases with the energy gradient, with a scaling exponent of about 0.29; ii – If the macroscale gradient is present, referring to the situation where the macroscale gradient is zero but the energy gradient is not, the intermittency is higher if the energy and scale gradients are concordant and is lower if they are opposite. The similarity analysis, that is in fair agreement with the previous experiments, is based on the use of the kinetic energy equation, which contains information concerning the third order moments of the velocity fluctuations. The analysis lies on two simplifying hypotheses: first, that the decays of the turbulences being mixed are nearly equal (as suggested by the experiments), second, that the pressure-velocity correlation is nearly proportional to the convective transport associated to the fluctuations (Yoshizawa, 2002).

The dependence of turbulence mixings on the initial conditions was considered and documented through single-point statistics, obtained by means of direct and large eddy numerical simulations (Tordella & Iovieno, 2004, Iovieno & Tordella, 2002). The simulations were carried out by means of a new technique for the parallel dealised pseudospectral integration of the Navier-Stokes equations (Iovieno et al., 2001). In all the shearless mixing experiments a self-similar state appeared to exist. The statistical distributions of orders higher than the second maintain features that depend on the initial values of the energy, $\mathcal{E}$, and macroscale, $\mathcal{L}$, ratios, and on the sign of $\mathcal{L}$. Independently of the values of the control parameters and the concurrency, or lack of it, of the energy and scale gradients, a set of common properties exists for all the studied mixings. First, the statistical distributions become self-similar after nearly a decay of three time units. Second, in the self-similarity region of the decay, the lateral spreading rate is on averege close to 0.15. Third, the kinetic energy distribution has a common shape (see, (8)). Fourth, all the mixings – including the mixing with $\mathcal{L} = 1$ are very intermittent, as the skewnes S and kurtosis K distributions show, see fig.s 1b and 2.

To carry out the similarity analysis we considered the second moment equations for the velocity fluctuations (u, in the inhomogeneous direction x, v_1, v_2 in the plane normal to x),

$$\begin{aligned}
\partial_t\overline{u^2} + \partial_x\overline{u^3} &= -2\rho^{-1}\partial_x\overline{pu} + 2\rho^{-1}\overline{p\partial_x u} - 2\varepsilon_u + \nu\partial_x^2\overline{u^2} &\quad (1)\\
\partial_t\overline{v_1^2} + \partial_x\overline{v_1^2 u} &= 2\rho^{-1}\overline{p\partial_{y_1} v_1} - 2\varepsilon_{v_1} + \nu\partial_x^2\overline{v_1^2} &\quad (2)\\
\partial_t\overline{v_2^2} + \partial_x\overline{v_2^2 u} &= 2\rho^{-1}\overline{p\partial_{y_2} v_2} - 2\varepsilon_{v_2} + \nu\partial_x^2\overline{v_2^2} &\quad (3)
\end{aligned}$$

The two mixed turbulences decay in a similar way, as shown by numerical simulations (Tordella & Iovieno, 2004). Thus, in the decay laws:

$$E_1(t) = A_1(t+t_0)^{-n_1}, \quad E_2(t) = A_2(t+t_0)^{-n_2}$$

the exponents n_1, n_2 are close each other. Here, we suppose $n_1 = n_2 = n = 1$, a value which corresponds to $R_\lambda \gg 1$ (Batchelor & Townsend, 1948).

In the absence of energy production, the pressure-velocity correlation has been shown to be proportional to the convective fluctuation transport (Yoshizawa, 1982, 2002)

$$-\overline{pu} = a\rho\frac{\overline{u^3} + 2\overline{v_1^2 u}}{2}, \quad a \approx 0.10,$$

so that

$$-\rho^{-1}\overline{pu} = \alpha\overline{u^3}, \quad \alpha = \frac{3a}{1+2a} \approx 0.25. \quad (4)$$

In this initial value problem, the moment distributions are determined by the coordinates x, t, and by the energy E and the macroscale ℓ of the two mixing turbulences. Thus

$$\overline{u^k} = E_1^{\frac{k}{2}}\varphi_{u^k}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}) \;\; \forall k, \;\; \varepsilon_u = E_1^{\frac{3}{2}}\ell^{-1}\varphi_{\varepsilon_u}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}), \quad (5)$$

where $\eta = x/\Delta(t)$, $\Delta(t)$ is the mixing layer thickness, $R_{\ell_1} = E_1^{\frac{1}{2}}(t)\ell_1(t)/\nu$ is the Reynolds number relevant to the high energy turbulence, $\vartheta_1 =$

$tE_1^{\frac{1}{2}}(t)/\ell_1(t)$ is the time scale of the flow, $\mathcal{E} = E_1(t)/E_2(t)$, $\mathcal{L} = \ell_1(t)/\ell_2(t)$ (where subscripts 1 and 2 refer to the high/low energy regions respectively). Notice that, if $n = 1$, $\mathcal{E}$, $\mathcal{L}$, $\vartheta_1 = n/f(R_{\lambda_1})$ and $R_{\ell_1} \propto t^{1-n}$ are constant. Inserting relations 5 in (1), it is deduced that $\Delta(t) \propto \ell_1(t)$. By putting $\Delta(t) = \ell(t)$, one obtains:

$$-\frac{1}{2}\eta\frac{\partial\varphi_{uu}}{\partial\eta} + \frac{1}{f(R_{\lambda_1})}(1-2\alpha)\frac{\partial\varphi_{uuu}}{\partial\eta} + \frac{\nu}{Af(R_{\lambda_1})^2}\frac{\partial^2\varphi_{uu}}{\partial\eta^2} = = \varphi_{uu} - \frac{2}{f(R_{\lambda_1})}\varphi_{\varepsilon_u} \qquad (6)$$

Given the lateral boundaries of the mixing, that correspond to homogeneous conditions for the turbulence, one observes that the **rhs** of (6) must be an odd function of η. The previously mentioned experiments suggest to model this **rhs** by means of a term proportional to $\partial_\eta^4\varphi_{uu}$ (superdiffusive behaviour, with β as constant of proportionality). In the following, by simply writing f instead of $f(R_{\lambda_1})$, the skewness, $S = \varphi_{uuu}/\varphi_{uu}^{3/2}$, reads

$$S = \frac{\varphi_{uu}^{-\frac{3}{2}}}{(1-2\alpha)}\left[\frac{f}{2}\int_{-\infty}^{\eta}\eta\frac{\partial\varphi_{uu}}{\partial\eta}\mathrm{d}\eta + \frac{\nu}{A_1 f}\frac{\partial\varphi_{uu}}{\partial\eta} + \beta f\frac{\partial^3\varphi_{uu}}{\partial\eta^3}\right] \qquad (7)$$

By representing the second moments with the fitting curve given by the experimental distributions (Veeravalli-Warhaft, 1989 and Tordella-Iovieno, 2004)

$$\varphi_{uu} = \frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta) \qquad (8)$$

one gets

$$S = \frac{1-\mathcal{E}^{-1}}{\sqrt{\pi}}\mathrm{e}^{-\eta^2}\left[\frac{f}{4(1-2\alpha)}\left(1-\frac{4\nu}{A_1 f^2}\right) + 2\beta(1-2\eta)\right] \times \left[\frac{1+\mathcal{E}^{-1}}{2} - \frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta)\right]^{-\frac{3}{2}} \qquad (9)$$

Figure 1 shows the good agreement of the modelled variance and skewness distributions (relations 8 and 9) with the experimental data. In fig. 2 the intermittency parameter associated to the lateral penetration of the mixing is compared to the values given by the present similarity law. It is observed that the scaling exponent deduced from the experiment (Tordella & Iovieno, 2004), which is approximately equal to 0.29, is correctly represented.

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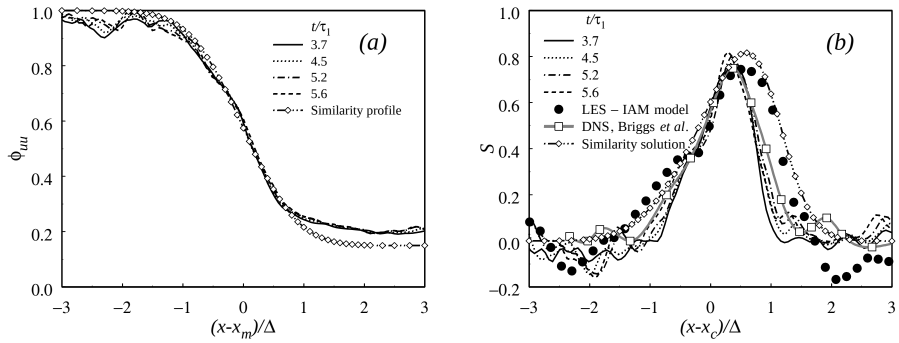

Figure 1: Normalized energy and skewness distributions; $\mathcal{E} = 6.7$ and $\mathcal{L} = 1$.

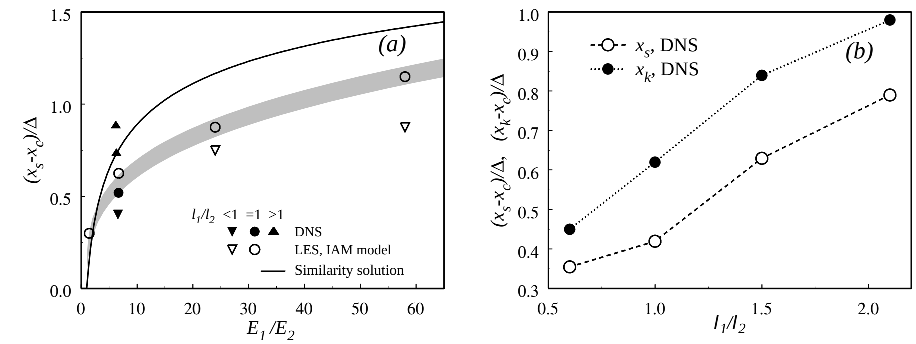

Figure 2: Position of the maximum of the skewness S and kurtosis K distributions as a function of the initial ratio of energy, part (a), and the initial ratio of integral scale, part (b): x_s and x_k are the positions of the maximum of $S(x)$ and $K(x)$, respectively.

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