Survival distribution of the stretching and tilting of vortical structures in isotropic turbulence. Anisotropic filtering analysis

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Vortex stretching probe function

The normalized stretching and tilting function

$$f(x,t) = \frac{|\omega \cdot \nabla u|}{|\omega|^2}(x,t)$$

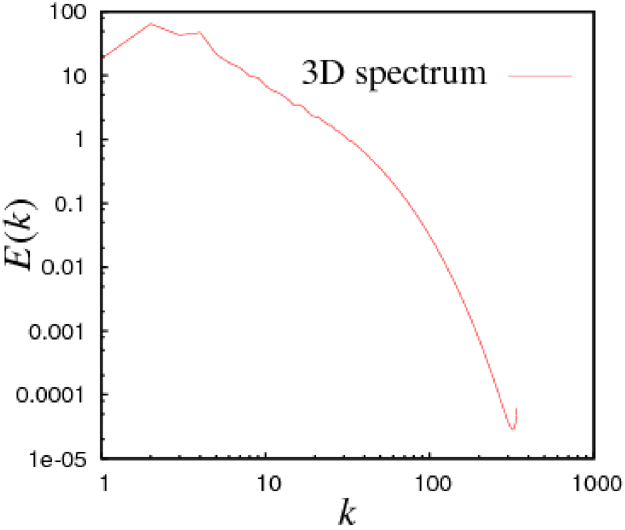
$$S(s) = P(f(x) > s)$$

Aim: analysis of statistical properties of the survival probability of the normalized stretching-tilting function to highlight its dependence on the different scale and structures of turbulence

- Stretching and tilting of vortical structures
- 3D inner scales formation
- Statistical properties of filtered fields

Large-eddy simulation of hypersonic flows. Selective procedure to activate the sub-grid model only where small scale turbulence is present (CPC 2007, CPC 2013, to appear).

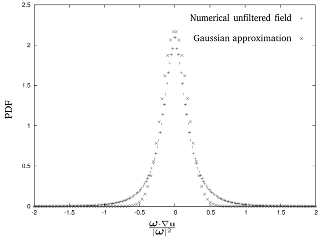
HIT Data Set, 3D spectrum

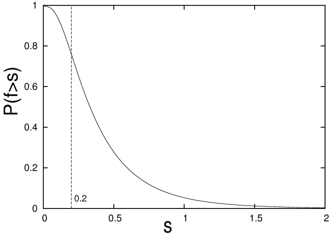


Biferale,Boffetta,Celani,Lanotte,Toschi, PoF 17, 2005.

PDF, survival distribution function

On the left: probability density function of one component of the vector $\frac{\omega \cdot \nabla u}{|\omega|^2}$ in an isotropic velocity field with $Re_\lambda = 280$. Comparison with the Gaussian model. The Skewness is negligible, about 10^{-2}. The Kurtosis however is very high and reaches a value of about 55.

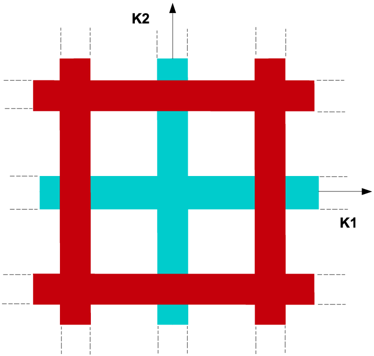




On the right: survival probability of the normalized stretching-tilting function in a fully resolved isotropic 3D turbulent field ($P(f(x) \geq s)$, $Re_\lambda = 280$. Unfiltered velocity field. The dashed vertical line indicates the value of f where the probability density function is maximum.

The probability of having a stretching-tilting larger than twice the local enstrophy is negligible, about 1 out of 10^3.

Filtering in wavenumber domain



Scheme of the filtering, projection in (k_1, k_2) wavenumbers plane. Wavenumbers along any possible direction that have at least one component below a threshold or inside a range are removed. We analyze these survival statistics when the large, the small inertial or the small inertial and dissipation scales are filtered out [to reduce the integral scale, Tordella & Iovieno, JFM 2006]:

$$g_{hp}(\underline{k}) = \prod_i \phi(k_i; k_{MIN}), \;\; \phi(k_i, k_{MIN}) = \frac{1}{1 + e^{-(k_i - k_{MIN})}}$$

or as a band-stop filter that reduces the contribution of the variable band:

$$k_{MIN} < k_1 < k_{MAX} \quad \text{or} \quad k_{MIN} < k_2 < k_{MAX} \text{ or } k_{MIN} < k_3 < k_{MAX}.$$

$$\begin{aligned} g_{bs}(\underline{k}) &= \prod_i \overline{\phi}(k_i; k_{MIN}, k_{MAX}), \\ \phi(k_i; k_0) &= \frac{1}{1 + e^{-(k_i - k_0)}}, \\ \overline{\phi}(k_i; k_{MIN}, k_{MAX}) &= [1 - \phi(k_i; k_{MIN})] + \phi(k_i; k_{MAX}) \end{aligned}$$

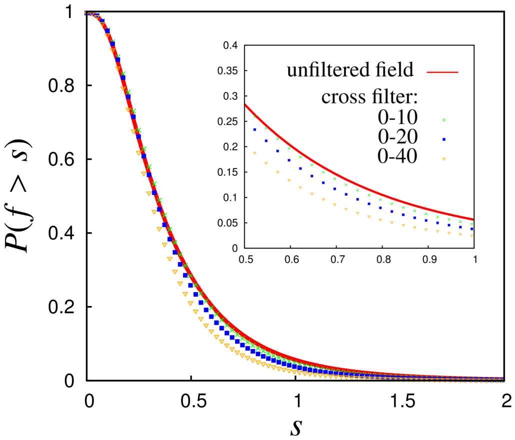
Analysis on filtered fields

Turbulent field parameters

- Data from DNS of Homogeneous and Isotropic Turbulence
- cubic domain, 1024^3 grid points
- $Re_\lambda = 280$
- root-mean-square velocity: $u_{rms} = 0.09$ m/sec
- viscosity: $\nu = 0.96 \cdot 10^{-6}$ m/s^2
- Taylor micro scale: $\lambda = 3$ mm
- integral scale: $\ell = 56$ mm

Analysis on filtered fields

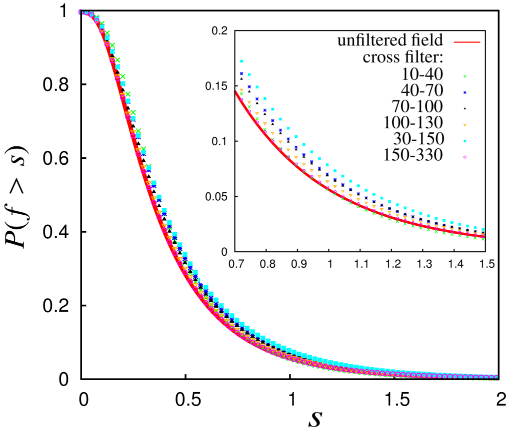
Survival distribution, high-pass filtering



Survival probability of the normalized stretching-tilting function in a high pass filtered isotropic turbulent field.

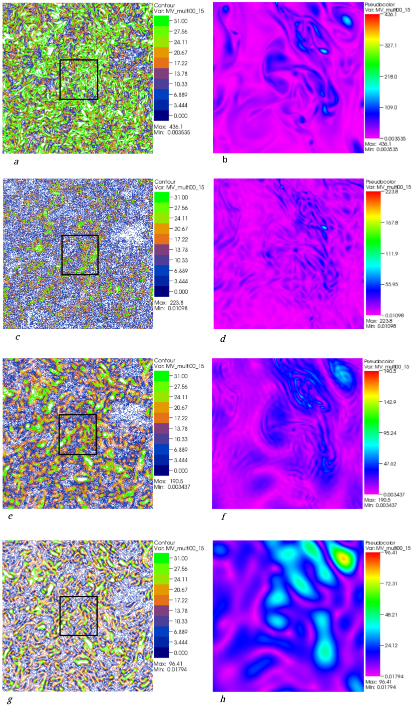
Analysis on filtered fields

Survival distribution, band-stop filtering



Probability of the normalized stretching-tilting function in a band-stop filtered isotropic turbulent field of being higher than a threshold s.

Control function: survival function $1 - F(x)$, band-stop filtering in various portion of the inertial and dissipative range.



First row (a,b): unfiltered field.

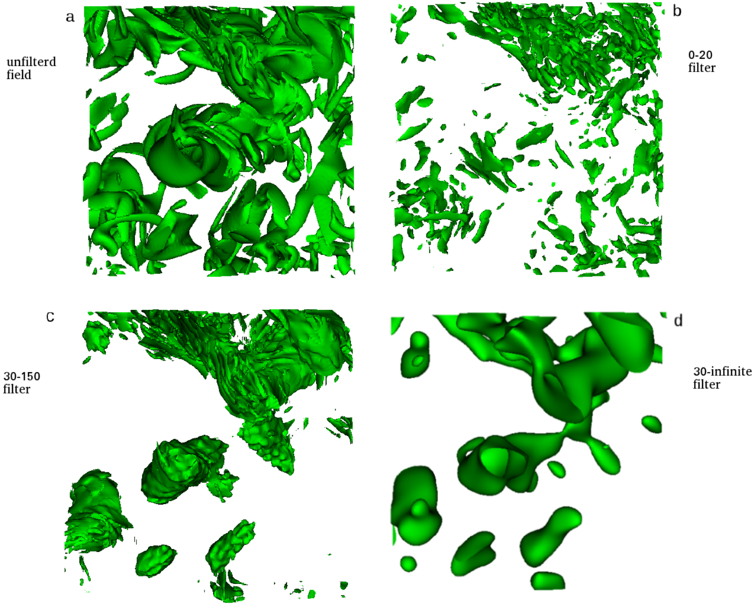
Second row (c,d): the wave number range 0-20 is filtered out by using the high-pass cross filter.

Third row (e,f): the wave number range 30-150 is filtered out by using the band-stop cross filter.

Fourth row (g,h) the wave number range 30-infinity is filtered out by using the low-pass cross filter.

Vortex stretching probe function
HIT Data Set
Statistics
CROSS Filtering
Analysis
Visualization
Alignement
Comments

Alignement



Enstrophy production

- $\sigma_i =< \omega^2 \lambda_i cos^2(e_\omega, e_i) >;$
- $\sigma_{tot} = \sum_i < \omega^2 \lambda_i cos^2(e_\omega, e_i) >$

TABLE I: normalized strain rate tensor eigenvalues and normalized enstrophy production computed in the unfiltered and the filtered field

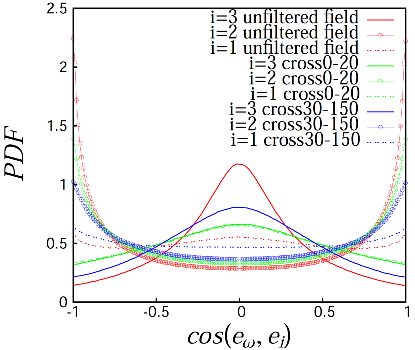
Principal index axes	Reference unfiltered field (Re_λ=280)	Large scales filtered (cross 0-20) *	Intermediate and small scales filtered (cross 30-150)**	Inertial and dissipative scales filtered cross(30-infinito) ***	unfiltered field ($Re_\lambda=10^4$) [1]
Normalized strain rate tensor eigenvalues $<\lambda_i>/(<\lambda_1^2+\lambda_2^2+\lambda_3^2>)^{0.5}$					
I	0.54	0.6121	0.57	0.56	0.47
II	0.12	0.0013	0.091	0.090	0.06
III	-0.66	-0.6108	-0.64	-0.64	-0.53
Normalized enstrophy production $<\sigma_i>/<\sigma_{tot}>$					
I	1.14	6.31	2.67	2.86	1.06
II	0.66	0.59	0.49	0.45	0.51
III	-0.80	-5.91	-2.16	-2.31	-0.57

* *the smallest 20 wavenumbers are filtered out*
** *wavenumbers in the inertial 30-150 range are filtered out*
*** *all wavenumbers above 30 are filtered out*

[1] M. Kholmyansky, A. Tsinober and S. Yorish, Physics of fluids, 13, 311 (2000).

- All filters increase the gap between the eigenvalue $< \lambda_1 >$ and $< \lambda_2 >$ and the gap between $< \lambda_2 >$ and $< \lambda_3 >$
- large scales missing induces $< \lambda_1 >$ and $< \lambda_3 >$ very close.

Vorticity and Strain tensor alignment



red: unfiltered field

green: wavenumbers 0-20 filtered out

blue: wavenumbers 30-150 filtered out

PDFs of the cosine of the angle between vorticity, ω, and the eigenvector, e_i, of the rate of strain tensor.

Comments

- In the unfiltered isotropic field, the probability of the ratio $|\omega \cdot \nabla u|/|\omega|^2$ being higher than a given threshold is higher than in the fields where the large scales were filtered out. At the same time, it is lower than in the fields were the small inertial and dissipation range of scales is filtered out.

Comments

- This seems to be basically due to the suppression of compact structures in the ranges that have been filtered in different ways. The partial removal of the background of filaments and sheets does not have a first order effect on these statistics.
- This study can be viewed as a kind of test for this idea and tries to highlight its limits. A qualitative relation in physical space and in Fourier space may be supposed to exist for blobs only. That is for the near isotropic structures which are sufficiently described by a single spatial scale and do not suffer from the disambiguation problem as filaments and sheets do.

Main results

- large scales contribute more to the Stretching-Tilting than to magnitude Vorticity
- inertial scales contribute more to magnitude Vorticity than to the Stretching-Tilting
- visualization of band-stop filtered field highlights small scale shape f(x)
- filters causes changes in the PDF value of the alignment between vorticity and eigenvector of the strain rate tensor.

THANK YOU