Localization of unresolved regions in the *selective* large-eddy simulation of hypersonic jets

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Summary. A method for the localization of the regions where the turbulent fluctuations are unresolved is applied to the *selective* large-eddy simulation (LES) of a compressible turbulent jet of Mach number equal to 5. This method is based on the introduction of a scalar probe function f which represents the magnitude of the *twisting-stretching* term normalized with the enstrophy [1]. The statistical analysis shows that, for a fully developed turbulent field of fluctuations, the probability that f is larger than 2 is zero, while, for an unresolved field, is finite. By computing f in each instantaneous realization of the simulation it is possible to locate the regions where the magnitude of the normalized stretching-twisting is anomalously high. This allows the identification of the regions where the subgrid model should be introduced into the governing equations (selective filtering).

The results of the selective LES are compared with those of a standard LES, where the subgrid terms are used in the whole domain. The comparison is carried out by assuming as high order reference field a higher resolution Euler simulation of the compressible jet. It is shown that the *selective* LES modifies the dynamic properties of the flow to a lesser extent with respect to the classical LES.

1 Small scale detection criterion

The regions where the fluctuations are unresolved are located by means of the scalar probe function (in the following called small scale localization criterion) [1]

$$f(\mathbf{u}, \boldsymbol{\omega}) = \frac{|\, (\boldsymbol{\omega} - \overline{\boldsymbol{\omega}}) \cdot \nabla(\mathbf{u} - \overline{\mathbf{u}}) \,|}{|\, \boldsymbol{\omega} - \overline{\boldsymbol{\omega}} \,|^2} \tag{1}$$

where $\mathbf{u}$ is the velocity vector, $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the vorticity vector and the overbar denoted the statistical average. Function (1) is a normalized scalar form of the vortex-stretching term that represents the inertial generation of three dimensional vortical small scales. When the flow is three dimensional and rich in small scales f is necessarily different from zero. In two-dimensional it is equal to zero. The mean flow is subtracted from the velocity and vorticity fields in order to consider the fluctuating part only of the field. The statistical distribution of f has been computed in a fully resolved turbulent fluctuation field (DNS of a homogenous and isotropic turbulent flow (1024^3, $Re_\lambda = 230$, data from [2])) and in some unresolved instances obtained by filtering this DNS field on coarser grids (from 512^3 to 64^3).

Figure 1(a) shows the probability that $f \geq t_\omega$ for all the resolutions considered: the probability that f assumes values larger a given threshold t_ω is always higher in the filtered fields and increases when the resolution is reduced. The difference between the probabilities in fully resolved and in filtered turbulence is maximum when $t_\omega \in [0.4, 0.5]$ for all resolutions, see figure 1(b). In such a range the probability $p(f \geq t_\omega)$ in the less resolved field is about twice the probability in the DNS field. This can lead to the introduction of a threshold t_ω of the values of f, such that, when f assumes larger values the field could be considered locally unresolved and should benefit from the local activation of the Large Eddy Simulation method (LES) by inserting a subgrid scale term in the motion equation. The values of this threshold can be chosen to be equal to that where the difference between the resolved and unresolved field is maximum, that is $t_\omega \approx 0.4$.

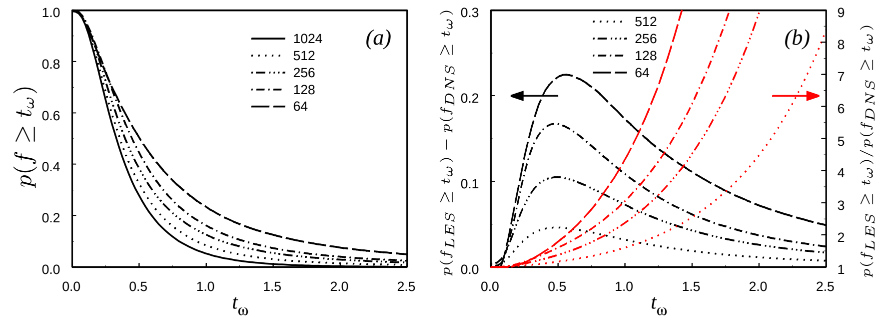

Fig. 1. (a) Distribution of the probability of having f for different resolutions of incompressible homogeneous and isotropic turbulence; (b) comparison between the probability in the filtered fields and in the unfiltered 1024^3 DNS field [2].

In order to investigate the presence of regions with anomalously high values of f, a set of tests have been carried out on existing Euler simulations of the temporal evolution of a perturbed, initially cylindrical, jet of initial $M = 5$ and density ratio between the ambient medium and the jet medium equal to 10, see section 2 and [1]. The regions where f assumes values larger than 0.4 have been filled in black in the visualizations shown in figure 2. According to the present criterion, these black regions can be viewed as regions where small scales are present and unresolved and where subgrid-scale terms should be introduced

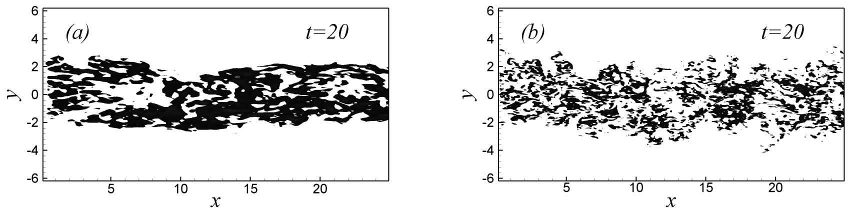

Fig. 2. Contour plots of the small scale function $f = 0.4$ in longitudinal sections $z = 0$ at dimensionless time $t = 20$: (a) simulation with 128^3 grid points, (b) simulation with 256^3 grid points.

into the governing equations. It should be noted, that this statistical analysis is funded on the Morkovin hypothesis that the compressibility effects do not have much influence on the turbulence dynamics, apart from varying the local fluid properties [4].

2 Results

We have studied numerically, and in Cartesian geometry, the temporal evolution of a 3D jet subject to periodicity conditions along the longitudinal direction. The flow is governed by the ideal fluid equations for mass, momentum, and energy conservation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\Big(\rho u_i\Big) = 0 \tag{2}$$

$$\frac{\partial (\rho u_k)}{\partial t} + \frac{\partial}{\partial x_i}\Big(\rho u_i u_k + p\delta_{ik}\Big) = \frac{\partial}{\partial x_i} H\left(f_{\mathrm{LES}} - t_\omega\right) \tau_{ik}^{SGS} \tag{3}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_i}\Big[(E+p)u_i\Big] = \frac{\partial}{\partial x_i} H(f_{\mathrm{LES}} - t_\omega) q_i^{SGS} \tag{4}$$

where the fluid variables p, ρ and u_i and E are, as customary, the pressure, density, velocity, and total energy respectively; and where τ_{ik}^{SGS} and q_i^{SGS} are the subgrid stress tensor and total enthalpy flow, respectively. $H(\cdot)$ is the Heaviside step function, thus the subgrid scale fluxes are applied only in the regions where $f > t_\omega$. The threshold t_ω is taken equal to 0.4, which is the value for which the maximum difference between the probability density function $p(f > t_\omega)$ between the filtered and unfiltered turbulence was observed (see fig.1). Diffusive terms have been neglected in equations (2-4) as in the very high Reynolds number free flow we are trying to simulate they will be smaller of both the subgrid terms and the numerical diffusivity of the scheme. The initial flow structure is a cylindrical jet in a parallelepiped domain, described by a cartesian coordinate system (x, y, z). The initial jet velocity is along the y-direction; its symmetry axis is defined by $(x = 0,\ z = 0)$. The initial jet velocity, at $t = 0$, is $V_y(x, z) = V_0$ within the jet, i.e. for $(x, z) \le R$, where R is the initial jet radius, and $V_y(x,z) = 0$ elsewhere. The initial density is set to $\rho(y,z) = \rho_0$ within the jet and $\rho(y,z) = \nu\rho_0$ outside, with ν the initial density ratio of the external medium to jet proper. Finally, we assume that the jet is initially in pressure equilibrium with its surroundings; for this reason, we assume an initially uniform pressure distribution p_0, see [3 for further details..

In the following, we will express lengths in units of the initial jet radius R, times in units of the sound crossing time of the radius R/c_0 (with $c_0 = \sqrt{\Gamma p_0/\rho_0}$), velocities in units of c_0 (thus coinciding with the initial Mach number), densities in units of ρ_0 and pressures in units of p_0.

The standard Smagorinsky model with $C_s = 0.1$ has been implemeted as subgrid model, the turbulent Prandtl number is taken equal to 1. Equations (2-4) have been solved using an evolution of the PLUTO code [5], which is a Godunov-type code that supplies a series of high-resolution shock-capturing schemes that are particularly suitable for the present application. In order to discretize the Euler equations, we chose a version of the Piecewise-Parabolic-Method (PPM), which is third order accurate in space and second order in time.

The domain size is a $4\pi \times 10\pi \times 4\pi$ parallelepiped, with y along the initial jet velocity, covered by a uniform cartesian grid with $128\times320\times128$ points (for the high resolution reference simulation the grid points were $256 \times 640 \times 256$). We have adopted periodic boundary conditions in direction y and outflow conditions in the other directions.

Two additional simulations have also been performed for comparison. A standard non selective LES where the subgrid model was introduced in the whole domain, which is obtained by putting $H \equiv 1$ in (2-4), and a higher resolution (640×256^2) Euler simulation obtained by putting $H \equiv 0$.

A visualization of the pressure field in a longitudinal section at $t = 36$ can be seen in figures 3(a-c) for the 3 cases (selective LES, classical LES, high resolution Euler simulation). The comparison shows the higher smoothing and small scale suppression produced by the non selective use of the subgrid model. The time evolution of the enstrophy distribution at two time instants far from the initial one is shown in figure 4 as a function of the distance from the centre of the jet. While the agreement between the enstrophy distribution obtained with the selective LES simulation and with the reference high resolution Euler simulation is very good, the non selective simulation damps out the vorticity magnitude in the very center of the jet and in the outer part, and introduces a spurious accumulation in the intermediate radial region. As a results, the vorticity dynamics is highly modified. This is also evident when observing the spectrum of the turbulent kinetic energy. Figure 5 shows the kinetic energy spectrum in the intermittent region between the jet core and the surrounding ambient. In the non selective LES, for $t = 28$, there is a concentration of energy in the low wavenumber region, which becomes even more pronounced for $t = 36$. This is consistent with the higher level of enstrophy seen in figure 4 at the same distance from the centre of the jet. Thus, we can observe that the selective introduction of the subgrid model yields distributions much closer,

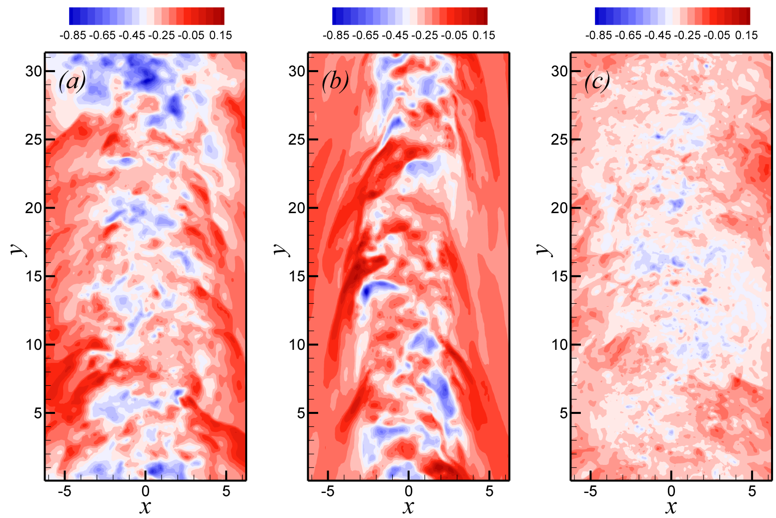

Fig. 3. Pressure distribution in a longitudinal section at $t = 36$: (a) selective LES, (b) standard LES, (c) higher resolution pseudo-DNS. The figures show the contour levels of $\log_{10}(p/p_0)$, the mean flow is from bottom to top.

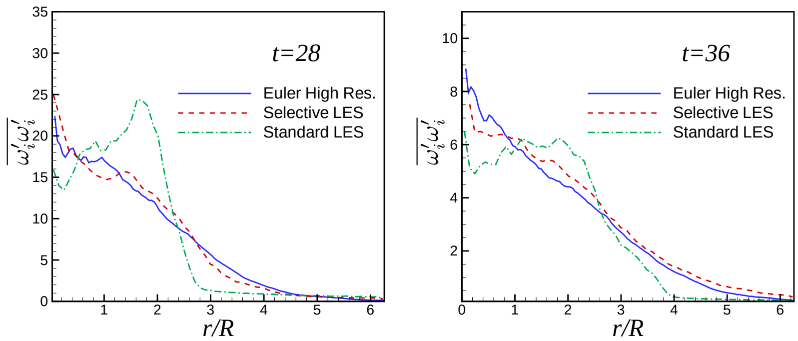

Fig. 4. Radial distribution of the enstrophy, r is the distance from the axis of the jet. All averages have been computed as space averages on cylinders.

with respect to the standard LES, to the distribution showns by the high resolution Euler simulation.

3 Concluding remarks

In this work we have shown that the *selective* LES, which is based on the use of a scalar probe function f – a function of the magnitude of the local stretching-twisting operator – can be conveniently applied to the simulation of compressible jets. The probe function f was coupled with the standard Smagorinsky subgrid model. However, it should be noted that the use of f can

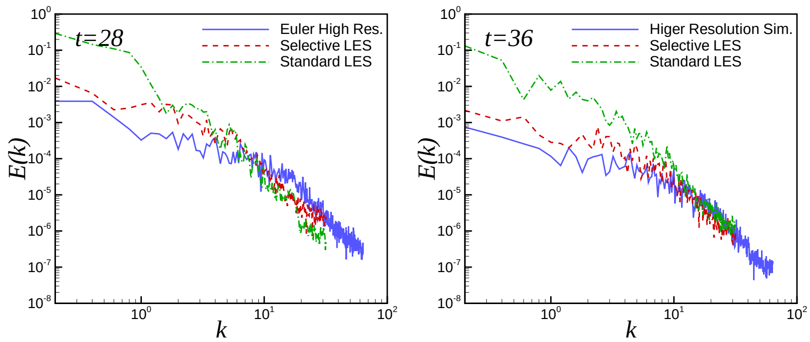

Fig. 5. Spectra of the turbulent kinetic energy at $r = 2$, computed as the Fourier transform of the two-point correlations of the fluctuating kinetic energy $\rho(u_i^2)/2$.

be coupled with any model because f simply acts as an independent switch for the introduction of a subgrid model. The comparison among the three kinds of simulation (selective LES, standard LES, high resolution reference) here carried out shows that this method can improve the dynamical properties of the simulated field, in particular, the spectral and vorticity distributions.

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